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## The Symbiotic Evolution of Millimeter-Wave Technology and Artificial Intelligence in the 6G Era

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### ABSTRACT

The sixth-generation (6G) of wireless communication, anticipated around 2030, promises a paradigm shift towards intelligent, hyper-connected services, extending far beyond the capabilities of current 5G networks. This article provides a comprehensive exploration of the symbiotic evolution of two critical enabling technologies for 6G: millimeter-Wave (mmWave) communications and Artificial Intelligence (AI). We delve into the fundamental characteristics and advancements in mmWave technology, highlighting its potential to unlock vast spectrum resources essential for 6G's ambitious data rate targets, alongside the inherent propagation challenges. The article then examines the pivotal role of AI as the engine for optimizing 6G network performance, detailing various AI techniques applicable to wireless communications and their specific use in enhancing mmWave systems through intelligent beam management, channel estimation, and radio resource management. Key Performance Metrics (KPIs) for AI-integrated 6G mmWave networks are discussed, encompassing both next-generation targets for traditional metrics like data rates, latency, and reliability, as well as novel metrics reflecting AI-native capabilities such as adaptability and learnability. The integration of AI across the 6G network architecture, including the Radio Access Network (RAN) and Core Network, is analyzed, supported by statistical insights and foundational mathematical models. Furthermore, the article explores transformative use cases enabled by this synergy, such as holographic communications, Extended Reality (XR), and intelligent infrastructure. Finally, we address the significant challenges related to complexity, scalability, energy efficiency, standardization, security, and ethical considerations, outlining crucial future research directions. This work concludes by synthesizing the indispensable roles of mmWave and AI in realizing the 6G promise and underscores the transformative potential of this evolution towards an intelligent and connected future.

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### الخلاصة

الجيل السادس (6G) من الاتصالات اللاسلكية، المتوقع ظهوره بحلول عام 2030، يعد بتحول جذري نحو خدمات ذكية فائقة الاتصال، تتجاوز بكثير قدرات شبكات الجيل الخامس (5G) الحالية. تقدم هذه المقالة استكشافاً شاملًا للتطور التكاملي بين تقنيتين أساسيتين تمكنان الجيل السادس: الاتصالات بمواجات المليمتر (mmWave) والذكاء الاصطناعي (AI). تتناول الخصائص الأساسية والتطورات في تقنية mmWave، مع تسليط الضوء على إمكاناتها في فتح موارد طيفية واسعة ضرورية لتحقيق أهداف الجيل السادس الطموحة في معدلات نقل البيانات، إلى جانب التحديات الطبيعية في الانتشار. ثم تستعرض المقالة الدور المحوري للذكاء الاصطناعي كمحرك لتحسين أداء شبكات الجيل السادس، مع شرح تقنيات الذكاء الاصطناعي المختلفة القابلة للتطبيق في الاتصالات اللاسلكية واستخدامها المحدد في تعزيز أنظمة mmWave من خلال الإدارة الذكية للحزام، وتقدير الفتوات، وإدارة الموارد الراديوية. تناقش مؤشرات الأداء الرئيسية (KPIs) لشبكات mmWave المدمجة بالذكاء الاصطناعي في الجيل السادس، وتشمل أهداف الجيل القادم للمؤشرات التقليدية مثل معدلات البيانات، وزمن الاستجابة، والموثوقية، بالإضافة إلى مؤشرات جديدة تعكس قدرات الذكاء الاصطناعي الأصلية مثل التكيف وقابلية التعلم. كما يتم تحليل دمج الذكاء الاصطناعي عبر بنية شبكة الجيل السادس، بما في ذلك شبكة الوصول الراديوية (RAN) والشبكة الأساسية، مدعومًا برأى إحصائية ونمذج رياضية أساسية.علاوة على ذلك، تستعرض المقالة حالات الاستخدام التحولية التي تتيحها هذه التوسيعات، مثل الاتصالات الهولوغرافية، الواقع الممتد(XR)، والبنية التحتية الذكية. وأخيراً، يتم تناول التحديات الكبيرة المتعلقة بالتعقيد، وقابلية التوسيع، وكفاءة الطاقة، والتزامن.

القياسي، والأمن، والاعتبارات الأخلاقية، مع تحديد اتجاهات البحث المستقبلية الحاسمة. تختتم هذه الدراسة بتلخيص الأدوار الحيوية لتقنيتي mmWave والذكاء الاصطناعي في تحقيق وع الجيل السادس، وتؤكد على الإمكانيات التحولية لهذا التطور نحو مستقبل ذكي ومتراoط.

## 1. INTRODUCTION: The Dawn of 6G and the Imperative for Innovation

The relentless evolution of wireless communication technologies has consistently reshaped societal interactions, economic landscapes, and technological frontiers (Fadhel, 2015). As the deployment of fifth-generation (5G) networks matures globally, the research community and industry stakeholders are already deeply engaged in conceptualizing and developing the sixth-generation (6G) of wireless systems. Anticipated to be commercially available around 2030, 6G is not envisioned as a mere incremental upgrade from 5G but as a transformative paradigm shift, promising to integrate the physical, digital, and biological worlds into a seamless, intelligent, and hyper-connected continuum (Fayad et al., 2024; Saoud et al., 2024). This new era of connectivity will be characterized by unprecedented performance metrics, novel service capabilities, and a profound reliance on emerging technologies, among which millimeter-Wave (mmWave) spectrum and Artificial Intelligence (AI) are poised to play pivotal roles. The imperative for innovation in 6G stems from the escalating demands of future applications, such as truly immersive extended reality (XR), holographic communications, massive-scale Internet of Things (IoT), autonomous systems, and sophisticated sensing services, all of which necessitate a network infrastructure that is not only faster and more reliable but also inherently intelligent and adaptive (Cui et al., 2025; Lloria et al., 2025; Ullah et al., 2025).

### 1.1 Defining the 6G Vision: Beyond Connectivity to Intelligent Services

The vision for 6G extends far beyond the traditional metrics of increased data rates and reduced latency, although these remain critical enablers. It encompasses a future where connectivity is ubiquitous, intelligent, and deeply intertwined with human activities and environmental perception. Key themes characterizing the 6G vision include the convergence of communication, computation, and sensing, leading to a network that can perceive its environment, learn from interactions, and proactively optimize its operations (SNS, 2021). This shift towards “intelligent services” implies that 6G networks will not just transmit data but will actively participate in data processing, decision-making, and service provisioning. The concept of a “network as a sensor” or “integrated sensing and communication (ISAC)” is a prominent aspect, where the network infrastructure itself becomes a distributed sensing platform, enabling high-resolution environmental awareness for applications ranging from autonomous driving to healthcare monitoring. Furthermore, 6G aims to deliver truly global coverage, including in remote and underserved areas, potentially leveraging non-terrestrial networks (NTNs) such as satellites and high-altitude platforms (HAPs). Sustainability, trustworthiness, and digital inclusion are also integral components of the 6G vision, emphasizing the need for energy-efficient operations, robust security and privacy mechanisms, and equitable access to the benefits of next-generation connectivity (SNS JU, 2025). The ambition is to create a human-centric network that enhances quality of life, fosters economic growth, and addresses societal challenges through intelligent and pervasive connectivity (Chai et al., 2025; Mehmood & Mehmood, 2025; Siddiky et al., 2025).

### 1.2 The Role of Millimeter-Wave (mmWave) in Unlocking 6G Potential

Millimeter-Wave (mmWave) frequencies, typically ranging from 30 GHz to 300 GHz, offer vast swathes of underutilized spectrum, which is crucial for achieving the multi-terabit per second (Tbps) data rates envisioned for 6G (Fayad, Cinkler, & Rak, 2024). While 5G has initiated the use of mmWave bands, 6G is expected to exploit these and even higher frequency bands (sub-THz or THz) more extensively to meet its ambitious capacity and throughput targets. The availability of large contiguous bandwidths in the mmWave spectrum directly translates to significantly higher data transmission capabilities, as dictated by fundamental communication principles like the Shannon-Hartley theorem. This makes mmWave an indispensable technology for supporting bandwidth-hungry 6G applications such as uncompressed high-definition video streaming, real-time holographic telepresence, and massive data uploads from distributed sensors. However, mmWave communication is not without its challenges. Signals at these high frequencies suffer from severe path loss, atmospheric absorption, and susceptibility to blockage by common materials, which can limit their propagation range and reliability. Overcoming these challenges necessitates advanced antenna technologies, such as massive Multiple-Input Multiple-Output (MIMO) and sophisticated beamforming techniques, to focus radio energy into narrow, steerable beams, thereby compensating for propagation losses and improving signal quality (Fayad, Cinkler, & Rak, 2024). The evolution of mmWave technology, coupled with intelligent network

management facilitated by AI, will be critical in harnessing its full potential to deliver the extreme performance required by 6G networks (Abdul-Wajid, 2025; Abou Yassin et al., 2025; Saeed et al., 2025; Yang et al., 2025).

## 2. Millimeter-Wave Communications: Fundamentals and Advancements for 6G

The quest for higher data rates and increased capacity in wireless networks has consistently driven the exploration of new spectrum frontiers. Millimeter-wave (mmWave) frequencies, spanning from 30 GHz to 300 GHz, represent a significant leap in this direction, offering unprecedented bandwidth availability compared to the congested sub-6 GHz bands traditionally used for mobile communications (Fayad, Cinkler, & Rak, 2024). While 5G systems have made initial forays into utilizing mmWave spectrum, 6G is poised to leverage these and potentially higher frequency bands (e.g., sub-Terahertz) even more extensively to realize its ambitious performance targets, including terabit-per-second data rates and ultra-low latency. The unique characteristics of mmWave propagation, however, present both substantial opportunities and formidable challenges that necessitate innovative technological solutions and intelligent network management, areas where Artificial Intelligence (AI) is expected to make significant contributions (Liu et al., 2025).

### 2.1. Characteristics and Propagation Challenges of mmWave Frequencies

Millimeter-wave signals possess very short wavelengths, which fundamentally influences their interaction with the environment. One of the most significant characteristics is the high free-space path loss, which increases quadratically with frequency (as per the Friis transmission equation). This means that, for a given transmission power and antenna gain, mmWave signals attenuate much more rapidly with distance compared to lower-frequency signals. Consequently, the coverage range of individual mmWave base stations is inherently smaller, leading to denser network deployments. Furthermore, mmWave signals are highly susceptible to atmospheric absorption, particularly by oxygen and water vapor, with specific absorption peaks at certain frequencies (e.g., around 60 GHz for oxygen). This atmospheric attenuation can further limit the effective communication range, especially in outdoor environments and during adverse weather conditions like rain, which causes significant scattering and absorption (Saoud et al., 2024). Another critical challenge is the high penetration loss through common building materials such as concrete, brick, and even foliage. Unlike sub-6 GHz signals that can readily penetrate walls, mmWave signals are often blocked or severely attenuated, making indoor coverage from outdoor base stations difficult and necessitating dedicated indoor mmWave access points or repeaters. These signals also exhibit quasi-optical behavior, meaning they are prone to blockage by obstacles, including human bodies, leading to link instability and requiring sophisticated mechanisms for maintaining connectivity, such as multi-path routing and rapid beam switching. The combination of high path loss, atmospheric absorption, penetration losses, and sensitivity to blockage underscores the complexity of designing robust and reliable mmWave communication systems for 6G (Dogra et al., 2020).

### 2.2. Enabling Technologies for mmWave in 6G: Beamforming and Massive MIMO

To counteract the severe propagation losses and other challenges associated with mmWave frequencies, advanced antenna technologies are indispensable. Beamforming and massive Multiple-Input Multiple-Output (MIMO) systems are cornerstone enabling technologies for effective mmWave communication in both 5G and future 6G networks (Fayad et al., 2024). Beamforming involves using antenna arrays to concentrate radiated power in a specific direction, creating narrow, high-gain beams pointed towards the intended receiver. This directional transmission significantly increases the received signal strength, thereby extending the communication range and improving link quality. The short wavelengths of mmWave signals allow for the integration of a large number of antenna elements into a physically small array, making highly directional beamforming feasible. Analog, digital, and hybrid beamforming architectures are employed, each with its own trade-offs in terms of performance, complexity, and power consumption. Massive MIMO takes this concept further by deploying antenna arrays with hundreds or even thousands of elements at the base station. This not only enables highly precise and adaptive beamforming but also supports spatial multiplexing, allowing multiple data streams to be transmitted simultaneously to one or more users in the same time-frequency resource, thereby dramatically increasing spectral efficiency and overall system capacity (Alsharif et al., 2022; Maier et al., 2021). For 6G, the evolution of massive MIMO is expected to include even larger antenna arrays, potentially leveraging new materials and metasurfaces (Reconfigurable Intelligent Surfaces - RIS) to further enhance beam control and coverage. The dynamic nature of the wireless channel and user mobility in mmWave

environments necessitates highly agile beam management, including initial beam acquisition, beam tracking, and rapid beam switching in case of blockage. This is where AI and machine learning techniques are becoming increasingly crucial, offering intelligent solutions for optimizing beamforming strategies in real-time, predicting channel variations, and ensuring seamless connectivity (Saoud et al., 2024). The synergy between advanced antenna systems like massive MIMO and AI-driven control mechanisms will be fundamental to unlocking the full potential of mmWave spectrum for 6G (Alsharif et al., 2022; Maier et al., 2021).

### 3. Artificial Intelligence: The Engine for Optimizing 6G Networks

The unprecedented complexity, scale, and stringent performance demands of 6G networks necessitate a paradigm shift from traditional, often reactive, network management approaches to proactive, predictive, and autonomous operations. Artificial Intelligence (AI), with its diverse set of techniques for learning, reasoning, and decision-making, is emerging as the core engine to drive this transformation, enabling the optimization of 6G systems across various layers and functionalities (Saoud et al., 2024). The integration of AI is not merely an add-on feature but a fundamental design principle for 6G, aiming to create an “AI-native” network that can intelligently adapt to dynamic conditions, manage vast resources efficiently, and deliver novel services with enhanced quality of experience. From the physical layer challenges in mmWave communications to the sophisticated service orchestration in the core network, AI offers powerful tools to address the inherent complexities and unlock the full potential of 6G technologies (T. Huang et al., 2019).

#### 3.1 Overview of AI Techniques Applicable to Wireless Communications

A broad spectrum of AI techniques is being explored and adapted for applications in wireless communications, particularly in the context of 6G. Machine Learning (ML), a subfield of AI, is at the forefront, encompassing supervised learning, unsupervised learning, and reinforcement learning. Supervised learning algorithms, such as Support Vector Machines (SVMs) and Neural Networks (NNs), can be trained on labeled datasets to perform tasks like channel estimation, signal detection, and interference classification. Deep Learning (DL), a class of ML algorithms using deep neural networks with multiple layers, has shown remarkable success in handling complex, high-dimensional data, making it suitable for tasks like advanced beamforming, end-to-end communication system design, and sophisticated anomaly detection (Singh, 2025). Unsupervised learning techniques, including clustering and dimensionality reduction, are valuable for identifying patterns in unlabeled network data, such as traffic profiling and user behavior analysis. Reinforcement Learning (RL), particularly Deep Reinforcement Learning (DRL), enables agents to learn optimal policies through interaction with the environment, making it a promising approach for dynamic resource allocation, intelligent mobility management, and autonomous network control in 6G (Saoud et al., 2024). Beyond these, other AI paradigms like federated learning (for privacy-preserving distributed model training), transfer learning (for leveraging knowledge from one task to another), and explainable AI (XAI) (for understanding and trusting AI decisions) are also gaining traction to address specific challenges in 6G network design and operation. The choice of AI technique often depends on the specific problem, the availability of data, computational constraints, and the desired level of autonomy and performance (Pennanen et al., 2024; Siddiky et al., 2024).

#### 3.2 AI for Radio Resource Management in 6G mmWave Systems

Radio Resource Management (RRM) is a critical function in wireless networks, responsible for the efficient allocation and utilization of scarce radio resources such as spectrum, power, and time slots. In 6G mmWave systems, RRM becomes significantly more complex due to the dynamic channel conditions, high user mobility, directional communication requirements, and the need to support diverse service requirements with varying Quality of Service (QoS) demands. AI, particularly ML and DRL, offers powerful solutions to tackle these RRM challenges. For instance, AI algorithms can be employed for intelligent spectrum sensing and dynamic spectrum sharing, enabling more efficient utilization of the vast mmWave bands. AI-driven power control mechanisms can optimize transmission power to minimize interference and conserve energy, which is crucial given the dense deployment of mmWave cells. In the context of beamforming, AI can facilitate real-time beam selection, tracking, and adaptation to ensure robust links in highly dynamic environments (Saoud et al., 2024). DRL agents can learn optimal resource allocation policies that adapt to changing network loads and user demands, outperforming traditional rule-based or optimization algorithms in complex scenarios. Furthermore, AI can enable predictive RRM by forecasting traffic patterns, user mobility, and channel quality, allowing the network to proactively allocate resources and prevent congestion or service degradation. The integration of AI into RRM functions is essential for maximizing the efficiency, capacity, and reliability of 6G mmWave systems, ensuring that the network can dynamically adapt to the ever-changing wireless environment and user needs (John et al., 2025; Mahesh et al., 2023).

Technique	Typical performance gain	Computational & energy cost	Data needs & training time	Best use cases
Classical/ML (2VM, RF)	Moderate improvement over rule-based	Low to moderate milli-od CPU, energy	Low to moderate land mining 67,10229 data	Link detection
Shallow Neural Nets	Moderate hamming distances often better than	High, GPU/cTPU. memory high energy	Moderate savings during training on labeled data	Link Dissemination with geluciu
Deep Reinforcement Learning (DRL)	High for dynamic environments aiming policies that improve	High, GPU/TPU memory high energy	High...many accesses to long training time	Link decisions CSJ based on produce
Transfer Learning	Moderate slightly faster than what is currently used	Collaborative high communication overhead.	Collaborative CB) tracks across UIEs	On-device beans replacement
Explainable AI/XAI methods	May slightly reduce what is currently used greatly reduces	Variable, depends on basic model	Safety critical decisions, auditing beam usage more	On-device been replacement for power saving

Figure 1: Comparison table of AI Techniques for mmWave Tasks

#### 4. AI-Powered Enhancements for mmWave Performance in 6G

The successful deployment and operation of millimeter-Wave (mmWave) communication in 6G hinges on overcoming its inherent propagation challenges, such as high path loss, susceptibility to blockage, and channel dynamicity (Quy et al., 2023). Artificial Intelligence (AI) offers a transformative toolkit to address these issues, providing intelligent mechanisms to enhance the performance, reliability, and efficiency of mmWave links. By leveraging AI's capabilities in pattern recognition, prediction, and real-time optimization, 6G networks can achieve robust and adaptive mmWave communication, paving the way for the realization of ultra-high data rates and seamless connectivity (Saoud et al., 2024). AI-powered enhancements span various aspects of mmWave systems, from sophisticated beam management to precise channel state information (CSI) acquisition and proactive interference mitigation.

##### 4.1 Intelligent Beam Management and Tracking in Dynamic mmWave Environments

Effective beam management is paramount in mmWave systems due to their reliance on narrow, directional beams to compensate for high propagation losses. This includes initial beam alignment (finding the best beam pair between transmitter and receiver), beam tracking (maintaining alignment as users move or the environment changes), and beam switching (selecting a new beam path if the current one is blocked or degrades). Traditional beam management techniques can be slow and inefficient in highly dynamic 6G environments with dense user populations and frequent blockages. AI, particularly machine learning (ML) and deep reinforcement learning (DRL), provides powerful solutions for intelligent beam management. For instance, ML algorithms can learn from historical beam measurement data, user location information (if available), and environmental context (e.g., from sensors or cameras) to predict optimal beam directions, significantly reducing the overhead associated with exhaustive beam sweeping (Fayad, Cinkler, & Rak, 2024). DRL agents can be trained to make real-time decisions on beam selection and tracking, adapting to instantaneous channel conditions and user mobility patterns to maximize signal strength and minimize interruptions. AI can also enable proactive beam switching by predicting potential blockages based on contextual information, allowing the network to establish an alternative link before the current one fails. Furthermore, AI techniques can optimize beam patterns themselves, shaping beams to minimize interference to other users or to cover specific areas more effectively. The integration of AI into beam management systems

transforms them from reactive to predictive and adaptive, ensuring robust and resilient mmWave connectivity in complex 6G scenarios (Quy et al., 2023).

#### 4.2 AI-driven Channel Estimation and Prediction for mmWave Links

Accurate channel state information (CSI) is crucial for optimizing various communication tasks, including beamforming, resource allocation, and interference management. However, acquiring precise CSI in mmWave systems is challenging due to the high dimensionality of massive MIMO channels, the rapid channel variations caused by mobility and blockages, and the overhead associated with transmitting pilot signals. AI offers innovative approaches to improve channel estimation and prediction. Deep learning models, such as Convolutional Neural Networks (CNNs) and Recurrent Neural Networks (RNNs), can learn complex channel characteristics from partial or noisy measurements, enabling more accurate and efficient CSI acquisition (Saoud et al., 2024). For example, AI can be used for channel fingerprinting, where a database of channel characteristics at different locations is created, and ML algorithms predict the current channel based on location information and other sensor data. AI can also enhance channel prediction by learning temporal and spatial correlations in channel variations, allowing the network to anticipate future channel states and proactively adapt transmission parameters. This is particularly important for maintaining link quality for mobile users and for enabling predictive resource allocation. Moreover, AI can assist in compressing CSI feedback from users to base stations, reducing overhead in massive MIMO systems. By providing more accurate and timely CSI, AI-driven channel estimation and prediction techniques significantly contribute to enhancing the overall performance and reliability of 6G mmWave links, enabling more efficient use of spectrum and improved quality of service (L. Zhang et al., 2019).

#### 4.3 A Conceptual Framework for AI-mmWave Integration in 6G Networks

To transcend the purely descriptive nature of existing surveys, this work introduces a three-dimensional taxonomy that classifies the convergence of Artificial Intelligence (AI) and millimeter-wave (mmWave) technologies in 6G systems. The framework is designed to capture where, how, and why AI intervenes across the mmWave communication stack (Abou Yassin et al., 2025; Chai et al., 2025; Cui et al., 2025; Lloria et al., 2025; Siddiky et al., 2025).

##### (A) Integration Layer Dimension Where AI Operates

###### 1. Physical-Layer Intelligence:

AI models enhance signal propagation, beamforming, and channel estimation by learning non-linear radio environments.

Examples: Deep learning-based channel prediction, reinforcement learning for beam tracking.

###### 2. Network-Layer Intelligence:

Focused on spectrum allocation, mobility management, and interference mitigation across cells.

Examples: Graph neural networks for resource sharing, federated learning for coordinated RRM.

###### 3. Application-Layer Intelligence:

Uses AI insights from user behavior and QoS demands to orchestrate network slicing and service provisioning.

Examples: Semantic communications, adaptive XR and holographic streaming.

##### (B) AI Function Dimension How AI Contributes

###### 1. Modeling and Prediction:

Data-driven estimation of channel states, traffic, or mobility patterns.

###### 2. Optimization and Decision-Making:

Reinforcement or evolutionary learning to tune system parameters (power, beam, scheduling) under constraints.

###### 3. Control and Adaptation:

Real-time adaptation to environmental or traffic changes; AI acts as a closed-loop controller.

##### (C) Deployment Hierarchy Dimension — Where the Intelligence Resides

###### 1. Device-Level AI:

Lightweight on-device models enabling fast beam alignment and user tracking.

###### 2. Edge-Level AI:

Cooperative intelligence across multiple access points or base stations with reduced latency.

## 3. Cloud/Core-Level AI:

Global optimization using large-scale network data for long-term policy updates and training.

(D) Cross-Dimensional Insight Each research work in AI-mmWave integration can be mapped as a tuple:

(Layer, Function, Deployment)

For example, Beam tracking using DRL at the edge → (Physical, Optimization, Edge).

This framework highlights research concentration areas and exposes underexplored combinations such as (Network, Modeling, Device) or (Application, Control, Edge).

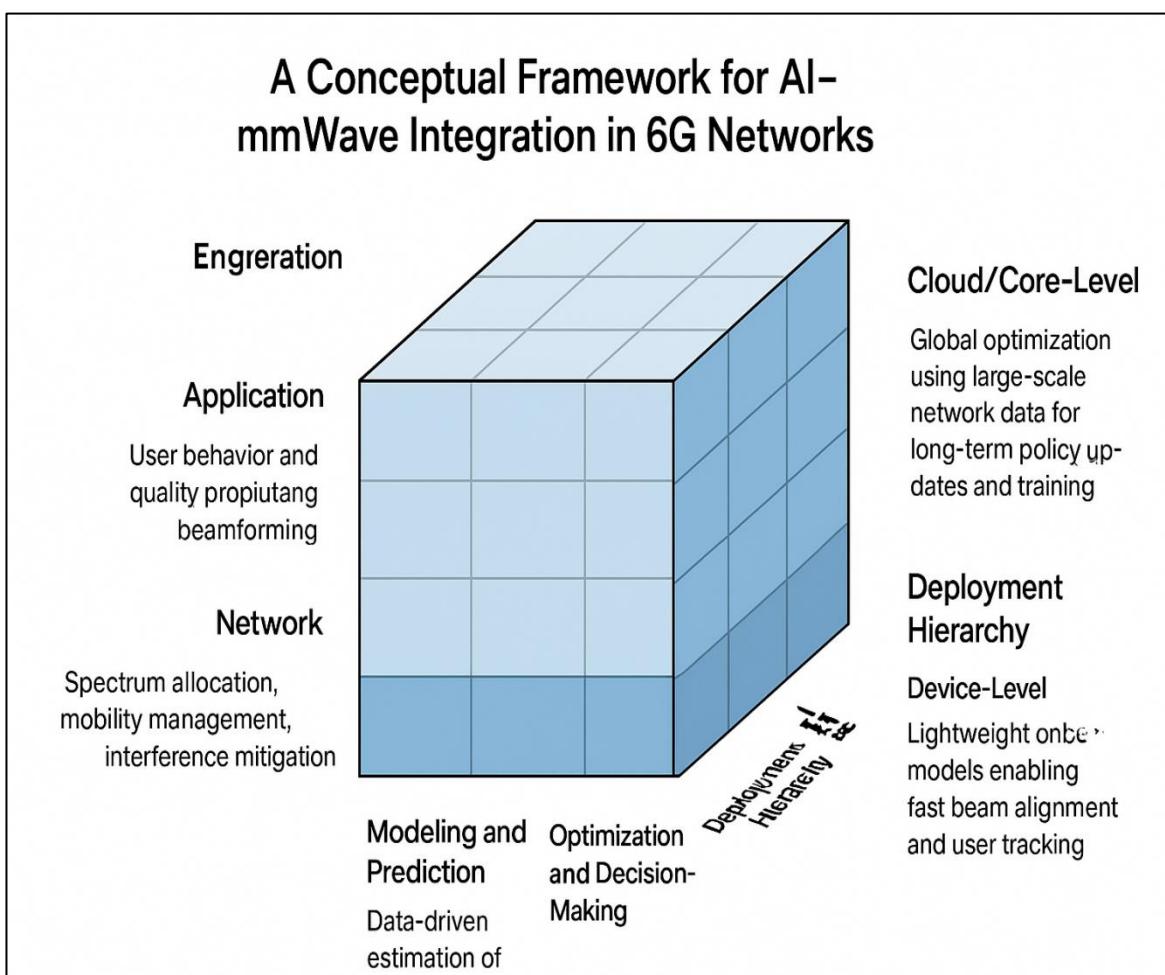
## (E) Significance of the Framework

This taxonomy provides:

- Comparative clarity: Easier benchmarking of AI solutions across layers.
- Gap identification: Highlights unaddressed AI roles or deployment levels.
- Research roadmap: Guides the design of integrated 6G architectures combining multiple intelligence layers.

Visual Representation A 3D cube diagram with the three axes:

- Integration Layer (x-axis)
- AI Function (y-axis)
- Deployment Hierarchy (z-axis)



**Figure 2:** A Conceptual Framework for AI–mmWave Integration in 6G Networks

## 5. Key Performance Metrics for AI-Integrated 6G mmWave Networks

The advent of 6G, with its profound integration of Artificial Intelligence (AI) and extensive use of millimeter-Wave (mmWave) spectrum, necessitates a re-evaluation and expansion of traditional wireless network performance metrics. While foundational metrics such as data rate, latency, and reliability remain crucial, the unique capabilities and complexities introduced by AI-native architectures and the specific characteristics of mmWave communications call for a more nuanced and comprehensive set of Key Performance Indicators (KPIs). These KPIs must not only quantify the raw performance enhancements but also capture the intelligence, adaptability, and efficiency that AI brings to 6G systems (SNS JU, 2025). The goal is to establish a framework for evaluating how effectively AI-integrated 6G mmWave networks can meet the diverse and demanding requirements of future applications, ranging from immersive XR to critical control systems (Khan et al., 2025).

### 5.1. Defining Next-Generation Performance: Data Rates, Latency, and Reliability Targets

The baseline performance expectations for 6G significantly surpass those of 5G, pushing the boundaries of what is technologically feasible. For data rates, 6G aims for peak throughputs in the order of Terabits per second (Tbps) and user-experienced data rates of Gigabits per second (Gbps) (SNS JU, 2025; Fayad, Cinkler, & Rak, 2024). These ultra-high speeds are essential for applications like holographic telepresence, real-time digital twins, and high-fidelity XR. The SNS JU White Paper (2025) outlines specific targets, such as a peak data rate potentially reaching 1 Tbps and a user-experienced data rate of 1 Gbps under various conditions. Latency is another critical metric, with 6G targeting end-to-end (E2E) latencies in the sub-millisecond range (e.g., 0.1 ms to 1 ms) for ultra-reliable low-latency communications (URLLC) use cases, such as industrial automation, remote surgery, and tactile internet applications (SNS JU, 2025). This represents a tenfold or greater reduction compared to 5G. Jitter, or latency variation, also becomes a critical KPI, especially for real-time services. Reliability and Availability targets are also exceptionally stringent, often aiming for “six nines” (99.9999%) or even higher availability for critical services, ensuring near-continuous connectivity and service uptime (SNS JU, 2025). This level of reliability is vital for safety-critical applications where network failures can have severe consequences. Other traditional KPIs, such as connection density (targeting up to 10 million devices per square kilometer), mobility (supporting speeds exceeding 500 km/h, potentially up to 1000 km/h for high-speed trains or aerial vehicles), and energy efficiency (aiming for a 10-100 fold improvement over 5G), are also being pushed to new limits by 6G (SNS JU, 2025). These ambitious targets for conventional KPIs form the foundation upon which the more AI-specific metrics are built (Dogra et al., 2020; Khan et al., 2025; Liu et al., 2025).

### 5.2. Novel Performance Metrics for AI-Native 6G Systems: Adaptability, Learnability, & Efficiency

Beyond the traditional KPIs, the deep integration of AI into 6G networks necessitates new metrics to quantify the performance and effectiveness of the embedded intelligence. The SNS JU White Paper (2025) highlights several AI-related capabilities and the need for corresponding KPIs. Adaptability refers to the network's ability to dynamically adjust its configuration and resource allocation in response to changing environmental conditions, traffic loads, user demands, or network faults. KPIs for adaptability might include the time taken to converge to an optimal state after a significant change, the range of conditions under which optimal performance can be maintained, or the reduction in human intervention required for network management. Learnability measures how quickly and effectively the AI models within the network can learn from new data and improve their performance over time. This could be quantified by the learning rate of AI algorithms, the accuracy improvement achieved with a given amount of training data, or the ability to generalize to unseen scenarios. Efficiency of AI Operations is also critical, encompassing metrics like the computational resources (e.g., processing power, memory) consumed by AI algorithms, the energy footprint of AI-driven network functions, and the processing time required for AI models to make decisions or predictions (SNS JU, 2025). For instance, an AI model for beam management might be evaluated not only on its accuracy but also on its inference latency and computational complexity. Furthermore, metrics related to explainability and trustworthiness of AI decisions will become increasingly important, especially for critical applications, to ensure that network operators can understand and rely on the autonomous actions taken by the AI. Other novel KPIs could include sensing accuracy and coverage (for ISAC capabilities), positioning accuracy and latency (for localization services), and context-awareness precision. These AI-centric KPIs, in conjunction with the enhanced traditional metrics, will provide a holistic view of the performance and

intelligence of 6G mmWave networks, guiding their design, optimization, and evolution (Iliev et al., 2021; Siddiky et al., 2025).

## 6. AI Integration Across the 6G Network Architecture

The transformative potential of Artificial Intelligence (AI) in 6G is not confined to specific functionalities like radio resource management or beamforming; rather, it envisages a pervasive integration of intelligence across the entire network architecture, from the edge to the core. This holistic approach aims to create a truly AI-native 6G system, where AI algorithms and models are embedded at various network layers and components, enabling end-to-end optimization, automation, and the delivery of novel, context-aware services (Saoud et al., 2024). The architectural integration of AI spans the Radio Access Network (RAN), where intelligent base stations and user equipment (UE) will operate, and the Core Network (CN), which will leverage AI for sophisticated functions like predictive resource allocation, dynamic network slicing, and enhanced security. This pervasive intelligence is key to managing the complexity and scale of 6G and to unlocking its full capabilities (Iliev et al., 2021; Liu et al., 2025).

### 6.1. AI in the Radio Access Network (RAN): Intelligent Base Stations and User Equipment

The 6G RAN is expected to be a highly dynamic and complex environment, characterized by ultra-dense deployments, the use of mmWave and higher frequency bands, massive MIMO systems, and diverse user requirements. AI will play a crucial role in optimizing RAN operations and enhancing performance at both the base station (gNB) and UE levels. Intelligent Base Stations will leverage AI for a multitude of tasks. As discussed earlier, AI-driven beam management, channel estimation, and interference mitigation will be critical for mmWave communications (Fayad, Cinkler, & Rak, 2024). Beyond these, AI can enable intelligent load balancing across cells, predictive handover management based on user mobility patterns and channel conditions, and dynamic cell shaping or sleeping to optimize coverage and energy consumption. AI algorithms can also facilitate self-organizing networks (SON) functionalities, allowing gNBs to autonomously configure, optimize, and heal themselves, reducing operational expenditure (OPEX). Intelligent User Equipment will also benefit from embedded AI. UEs can use AI for tasks like intelligent band selection, adaptive power control to prolong battery life, and local context awareness to request appropriate network services. AI at the UE can also assist in improving uplink transmission, for example, by predicting channel quality or selecting optimal transmission parameters. Furthermore, federated learning approaches can allow UEs to collaboratively train AI models without sharing their raw data, preserving privacy while contributing to global model improvement for tasks like traffic prediction or anomaly detection. The synergy between AI at the gNB and AI at the UE will create a more responsive, efficient, and personalized RAN experience in 6G (Rao et al., 2024).

### 6.2. AI in the Core Network: Predictive Resource Allocation and Network Slicing

6G Core Network (CN) will be responsible for managing network-wide resources, orchestrating services, and ensuring end-to-end quality of service for a vast array of diverse applications. AI integration in the CN is essential for handling this complexity and enabling advanced functionalities. Predictive Resource Allocation is a key area where AI can provide significant benefits. By analyzing historical traffic data, user behavior patterns, and contextual information, AI models can forecast future resource demands across different network segments and services. This allows the CN to proactively allocate resources (e.g., compute, storage, bandwidth) to prevent congestion, minimize latency, and ensure that service level agreements (SLAs) are met. This is particularly important for dynamic network slicing, where different logical network slices are created to cater to specific service requirements (e.g., eMBB, URLLC, mMTC). AI-driven Network Slicing can automate the lifecycle management of network slices, including their creation, scaling, and termination, based on real-time demand and performance monitoring (SNS JU, 2025). AI can optimize resource allocation within and across slices, ensuring efficient utilization of network infrastructure while guaranteeing isolation and performance for each slice. Furthermore, AI can enhance CN security by enabling intelligent threat detection, anomaly identification (Singh, 2025), and automated response mechanisms. AI can also play a role in optimizing routing paths, managing network function virtualization (NFV) infrastructure, and providing insights for long-term network planning and evolution. The intelligence embedded in the 6G CN will be crucial for creating a flexible, programmable, and highly automated network capable of supporting the diverse and dynamic service landscape of the future (Hong et al., 2021; Maier et al., 2021).

## 7. Statistical Insights and Mathematical Models for 6G mmWave with AI

The development and optimization of AI-integrated 6G mmWave networks rely heavily on a robust understanding of their expected performance, underpinned by statistical analysis and sound mathematical modeling. This section delves into the projected performance gains achievable through AI, supported by

statistical data, and outlines some of the foundational mathematical equations that model AI-driven mmWave optimization (Liu et al., 2025). The synergy between empirical data, statistical projections, and theoretical models is crucial for guiding research, development, and standardization efforts in the 6G era. The insights from sources like the SNS JU White Paper (2025) on 6G KPIs provide a quantitative basis for these discussions, while fundamental communication and AI theories offer the mathematical framework.

### 7.1. Projected Performance Gains: Statistical Analysis of AI Impact on 6G KPIs

The integration of AI is anticipated to yield substantial improvements across a wide range of 6G Key Performance Indicators (KPIs). Statistical projections, often derived from simulations, testbed experiments, and extrapolations from current AI applications in 5G, paint a compelling picture of AI's impact. For instance, in data rates and capacity, AI-driven dynamic spectrum management, intelligent beamforming, and interference coordination are projected to enhance spectral efficiency significantly. While specific quantifiable gains are still a subject of ongoing research, improvements in user-experienced data rates and overall system capacity are expected to be substantial, helping to achieve the target of 1 Gbps user-experienced data rate and 1 Tbps peak data rates (SNS JU, 2025). In terms of latency, AI-powered predictive resource allocation, proactive mobility management, and optimized scheduling algorithms can contribute to reducing end-to-end latency towards the sub-millisecond targets. For example, AI can predict network congestion or link degradation and reroute traffic or adjust resources proactively, minimizing delays. Singh (2025) highlights AI's role in speeding up real-time detection and response in network security, which has analogous benefits for latency-sensitive communication. Regarding reliability and availability, AI-based anomaly detection, fault prediction, and self-healing mechanisms are expected to improve network resilience. Studies like Singh (2025) show AI can boost anomaly detection rates by nearly 30% and reduce false alerts by about 25% in specific network contexts, which translates to more reliable operations. Similar gains are anticipated in maintaining the stringent 99.9999%+ availability targets for critical 6G services. Furthermore, AI is projected to enhance energy efficiency by optimizing power usage in base stations and user devices through intelligent sleep modes, adaptive power control, and optimized computational load distribution for AI tasks themselves (SNS JU, 2025). Statistical analysis of these projected gains, often presented in research papers and industry white papers, provides crucial benchmarks for evaluating the effectiveness of different AI strategies and for justifying the investment in AI-native 6G architectures.

### 7.2. AI-Driven Modeling and Optimization of Non-Linear mmWave/6G Systems

The optimization of 6G mmWave systems using AI is grounded in various mathematical principles and models. While a comprehensive list is extensive, some foundational equations illustrate the underlying concepts. The Shannon-Hartley Theorem remains a fundamental benchmark for channel capacity (Mahesh et al., 2023):

$$C = B * \log_2(1 + S/N)$$

Where  $C$  is the channel capacity,  $B$  is the bandwidth, and  $S/N$  is the signal-to-noise ratio. AI algorithms aim to optimize parameters that influence  $B$  (e.g., dynamic spectrum access) or  $S/N$  (e.g., intelligent beamforming to maximize  $S$ , interference mitigation to reduce  $N$ ). For mmWave channel modeling, path loss equations are critical. A common model is (Khan et al., 2025):

$$PL(d) [dB] = PL(d0) + 10 * n * \log_{10}(d/d0) + Xg$$

Where  $PL(d)$  is the path loss at distance  $d$ ,  $d0$  is a reference distance,  $n$  is the path loss exponent (which varies significantly for mmWave and depends on the environment), and  $Xg$  is a term for shadowing. AI can help in accurately estimating ' $n$ ' or predicting  $Xg$  based on environmental context. In AI-driven beamforming, optimization problems are often formulated. For example, the objective might be to maximize the received signal strength at the UE, which can be expressed as maximizing  $|h^H * w|^2$ , where  $h$  is the channel vector and  $w$  is the beamforming weight vector. AI algorithms, particularly DRL, learn policies to find the optimal ' $w$ ' in dynamic environments. The core of many machine learning algorithms involves minimizing a loss function. For instance, in supervised learning for channel estimation, the loss function  $L$  might be the Mean Squared Error (MSE) between the predicted channel state and the actual channel state (L. Zhang et al., 2019):

$$L = (1/M) * \sum (h_{pred} - h_{actual})^2$$

Where  $M$  is the number of samples. Neural networks use gradient descent or its variants to minimize such loss functions by adjusting network weights. For resource allocation, AI might solve complex optimization problems, often formulated with an objective function (e.g., maximizing sum-rate, minimizing latency) subject to constraints (e.g., power limits, QoS requirements). These mathematical foundations, combined with

statistical data from network operations, enable AI to learn, adapt, and optimize 6G mmWave systems effectively (Pennanen et al., 2024; L. Zhang et al., 2019).

Traditional analytical models—such as Shannon's capacity  $C = B \log_2(1 + \text{SNR})$  and log-distance path-loss  $PL(d) = PL_0 + 10n \log_{10}(d/d_0)$ —provide tractable but idealized descriptions of wireless links. However, real 6G mmWave/sub-THz channels exhibit severe non-linearities caused by multipath clustering, dynamic blockages, beam misalignment, and hardware impairments (e.g., phase-noise, non-linear power amplifiers). To cope with these complexities, AI models can learn or optimize system behavior directly from data, complementing or replacing closed-form expressions (B. Huang et al., 2025; Jin et al., 2022; Lavdas et al., 2023; Xue et al., 2024; Y. Zhang et al., 2024).

(a) Learning complex channel and propagation models

Deep networks  $f_\theta(\mathbf{x})$  can approximate the mapping from environmental/contextual features  $\mathbf{x}$  (e.g., position, orientation, material maps) to channel responses  $\mathbf{h}$ :

$$\hat{\mathbf{h}} = f_\theta(\mathbf{x}), \min_{\theta} \|\mathbf{h} - f_\theta(\mathbf{x})\|_2^2.$$

Unlike simple path-loss models,  $f_\theta$  captures non-linear effects such as scattering and blockage. Generative models (GANs, diffusion models) further synthesize spatial-temporal channel samples to augment scarce measurement data.

(b) Learning to approximate end-to-end system mappings

Instead of separately modeling each layer, AI can learn a direct mapping from system parameters to performance metrics, such as throughput or latency:

$$\hat{C} = g_\phi(\mathbf{p}), \mathbf{p} = [B, P_t, N_t, N_r, d, \theta, \dots],$$

where  $g_\phi$  replaces the analytical capacity formula with a learned surrogate that remains differentiable and can be embedded in optimization loops. This supports AI-based digital twins that emulate the wireless environment in real time.

(c) AI-based optimization of non-convex objectives

Beamforming, power control, and resource allocation in 6G are typically non-convex, high-dimensional problems:

$$\max_{\mathbf{w}} R(\mathbf{w}) = \log_2 \left( 1 + \frac{|\mathbf{h}^H \mathbf{w}|^2}{\sigma^2} \right), \text{s.t. } \|\mathbf{w}\|^2 \leq P_t.$$

Reinforcement learning or neural approximators can learn policies  $\pi_\theta(\text{state}) \rightarrow \mathbf{w}$  that approach or surpass heuristic solvers, especially under dynamic channel conditions where gradient information is unavailable.

(d) Hybrid model-driven + data-driven approaches

Physics-informed neural networks (PINNs) or model-driven deep unfolding combine known equations with trainable components:

$$\mathbf{h}^{(k+1)} = \mathbf{h}^{(k)} - \alpha_k \nabla_{\mathbf{h}} (L_{\text{phys}}(\mathbf{h}) + L_{\text{data}}(\mathbf{h}; \theta)),$$

ensuring consistency with physical laws while capturing residual non-linearities that classical models miss. This balances interpretability, generalization, and data efficiency.

(e) Performance-driven learning objectives

AI models can optimize utility functions directly:

$$\max_{\theta} \mathbb{E}_{\mathbf{s}} [U(\mathbf{s}, f_\theta(\mathbf{s}))],$$

where  $U$  may encode throughput-energy-latency trade-offs, fairness, or QoS constraints. Multi-objective learning frameworks or evolutionary algorithms handle competing KPIs.

(f) Interpretation and physical insight

Explainable-AI tools (e.g., SHAP, saliency maps) reveal which input features dominate learned models, helping engineers derive new empirical formulations or simplified semi-analytical approximations suitable for standards work.

#### Left Panel – *Classical Modeling*

Inputs: Bandwidth B, Tx power, distance d → Equations (Shannon, PL) → Output throughput

Visual: icons of formulas and antennas, deterministic arrows

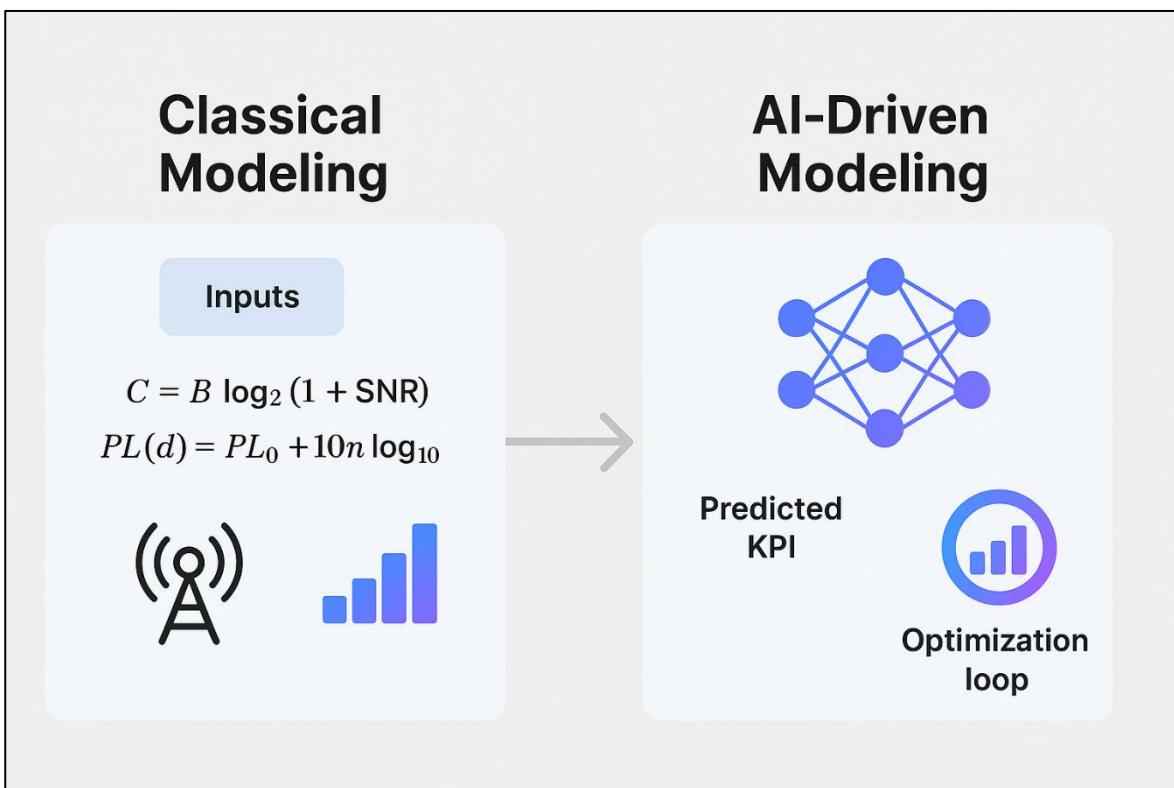
Color scheme: light gray & blue

#### Right Panel – *AI-Driven Modeling*

Inputs + environment/context → Neural model → Predicted KPI + Optimization loop

Visual: neural-network graph, loop arrow indicating self-learning/optimization

Color scheme: modern gradient (cyan → violet), signaling intelligence



**Figure 3:** From Analytical to AI-Driven Modeling of mmWave/6G Systems

## 8. Use Cases and Applications Enabled by AI in 6G mmWave Systems

The convergence of Artificial Intelligence (AI) with the vast bandwidth of millimeter-Wave (mmWave) spectrum in 6G networks is set to unlock a plethora of transformative use cases and applications that were previously confined to the realm of science fiction. These applications will leverage the ultra-high data rates, extremely low latency, massive connectivity, and inherent intelligence of 6G to create deeply immersive experiences, enable sophisticated autonomous systems, and revolutionize various industries (Saoud et al., 2024). The ability of AI to manage the complexities of mmWave communication and to extract meaningful insights from the data traversing the network is a critical enabler for these futuristic services. From deeply engaging holographic communications to the seamless operation of city-wide intelligent infrastructure, AI-driven 6G mmWave systems will redefine how humans interact with the digital and physical worlds.

### 8.1. Immersive Experiences: Holographic Communications and Extended Reality (XR)

One of the most anticipated application domains for 6G is the realm of immersive experiences, encompassing holographic communications, augmented reality (AR), virtual reality (VR), and mixed reality (MR)—collectively known as Extended Reality (XR). Holographic communications aim to transmit high-

fidelity, three-dimensional representations of people and objects in real-time, enabling truly immersive telepresence and remote collaboration. This requires enormous bandwidth (potentially terabits per second for high-resolution holograms) and extremely low latency (sub-millisecond) to ensure a seamless and natural interaction, demands that 6G mmWave is uniquely positioned to meet (SNS JU, 2025). AI will play a crucial role in compressing and decompressing holographic data, optimizing transmission over dynamic mmWave channels, and rendering complex 3D scenes efficiently. Extended Reality (XR) applications, which overlay digital information onto the physical world or create fully immersive virtual environments, will also be significantly enhanced by AI-integrated 6G. AI can personalize XR experiences, optimize rendering based on user gaze and context, and enable more natural interactions through voice and gesture recognition (SNS JU, 2025). The high data rates of mmWave will support streaming of high-resolution XR content, while low latency will minimize motion-to-photon delay, crucial for preventing cybersickness and ensuring a realistic experience. AI will also be vital for managing the massive data flows and computational loads associated with widespread XR adoption, ensuring consistent quality of service across numerous users (Maier et al., 2021; Pennanen et al., 2024).

## 8.2. Intelligent Infrastructure: Smart Cities, Autonomous Systems, and Industrial IoT

AI-driven 6G mmWave networks will form the backbone of future intelligent infrastructure, enabling a new generation of smart city services, autonomous systems, and advanced Industrial Internet of Things (IIoT) applications. In Smart Cities, 6G will connect a vast ecosystem of sensors, devices, and vehicles, generating massive amounts of data. AI will be essential for processing this data to optimize urban services such as intelligent transportation systems (ITS), smart energy grids, public safety, and environmental monitoring (Saoud et al., 2024). For example, AI can analyze real-time traffic data from mmWave-connected vehicles and sensors to optimize traffic flow, reduce congestion, and enhance road safety. Autonomous Systems, including autonomous vehicles, drones, and robots, rely on continuous, high-reliability, low-latency communication for navigation, coordination, and remote operation. 6G mmWave, enhanced by AI-driven beam management and predictive connectivity, will provide the robust communication links necessary for safe and efficient autonomous operations. AI algorithms will also process sensor data from these autonomous systems, enabling them to perceive their environment, make intelligent decisions, and collaborate effectively. In the Industrial IoT (IIoT) domain, 6G will support advanced manufacturing processes, such as digital twins, predictive maintenance, and real-time process control. AI will analyze data from industrial sensors to optimize production lines, predict equipment failures before they occur (Singh, 2025), and enable highly flexible and reconfigurable manufacturing environments. The precise positioning capabilities of 6G mmWave, further enhanced by AI, will also be crucial for tracking assets and guiding robots in industrial settings. These applications highlight how AI and 6G mmWave will synergize to create more efficient, responsive, and intelligent infrastructure across various sectors (Kebede et al., 2021; Sun et al., 2025; Zamanipour, 2019; Zhu et al., 2024).

## 9. Challenges and Future Research Directions

While the integration of Artificial Intelligence (AI) with millimeter-Wave (mmWave) technology in 6G networks promises a future of unprecedented connectivity and intelligent services, the path to realizing this vision is fraught with significant challenges. Addressing these hurdles and exploring new research frontiers will be crucial for the successful deployment and evolution of AI-driven 6G systems. The challenges span technological complexity, scalability, energy efficiency, standardization, security, and ethical considerations, each requiring concerted efforts from the research community, industry, and policymakers (Fayad, Cinkler, & Rak, 2024; Saoud et al., 2024).

### 9.1. Addressing Complexity, Scalability, and Energy Efficiency of AI in 6G

The sheer complexity of managing AI models within the vast and dynamic 6G ecosystem is a primary challenge. Training, deploying, and maintaining sophisticated AI algorithms across a distributed network infrastructure, from the core to the edge and end-user devices, requires robust MLOps (Machine Learning Operations) frameworks tailored for telecommunications. Ensuring the interoperability of AI models from different vendors and managing their lifecycle (updates, retraining, retirement) in a seamless manner is a non-trivial task. Scalability is another major concern. As the number of connected devices, users, and services in 6G networks grows exponentially, AI systems must be able to scale efficiently to handle the massive influx of data and computational demands without performance degradation. This includes scaling the training data pipelines, the inference capabilities at the edge and in the cloud, and the communication overhead associated with distributed AI. Energy efficiency is a critical challenge, particularly given the sustainability goals of 6G (SNS JU, 2025). AI algorithms, especially deep learning models, can be computationally intensive and power-hungry. Optimizing the energy consumption of AI processing at both the hardware and software levels, developing lightweight AI models suitable for resource-constrained devices, and designing energy-aware resource allocation for AI tasks are vital research areas. Future research should focus on developing novel AI

architectures that are inherently more efficient, exploring neuromorphic computing, and creating green AI solutions specifically for 6G networks (Mahesh et al., 2023).

## 9.2. Standardization, Security, and Ethical Considerations for AI-driven 6G

Standardization is essential for ensuring global interoperability and fostering a competitive ecosystem for AI-driven 6G. This includes standardizing interfaces for AI model exchange, data formats for training and inference, and performance evaluation methodologies for AI-based network functions. Organizations like ITU, 3GPP, and ETSI are actively working on these aspects, but consensus and timely standards development remain challenging given the rapid pace of AI innovation. Security in AI-driven 6G networks presents a multifaceted challenge. AI models themselves can be vulnerable to adversarial attacks (e.g., data poisoning, evasion attacks) that can compromise network performance or security. Conversely, AI can be a powerful tool for enhancing network security through intelligent threat detection and response (Singh, 2025). However, ensuring the robustness and resilience of both the AI systems and the network against sophisticated cyber threats is a critical research direction. This includes developing secure AI algorithms, robust defenses against adversarial machine learning, and privacy-preserving AI techniques (e.g., federated learning, homomorphic encryption) to protect sensitive user and network data. Ethical considerations are paramount as AI becomes more deeply embedded in communication networks that underpin many aspects of society. Issues such as algorithmic bias (e.g., unfair resource allocation or service discrimination), lack of transparency in AI decision-making (the “black box” problem), accountability for AI-induced errors or failures, and the potential for misuse of AI-powered surveillance capabilities need careful consideration and proactive governance. Future research must focus on developing explainable AI (XAI) techniques, fairness-aware AI algorithms, and robust ethical guidelines and regulatory frameworks to ensure that AI in 6G is deployed responsibly and for the benefit of all users (AI Kassir et al., 2022; Biliaminu et al., 2024; Q. Zhang & Wang, 2022).

## 10. Conclusion: Charting the Path Towards an Intelligent and Connected Future

The journey towards the sixth-generation (6G) of wireless communication represents a monumental leap forward, promising not just an evolution of existing capabilities but a revolution in how we connect, compute, and interact with the world. At the heart of this transformation lies the symbiotic relationship between advanced millimeter-Wave (mmWave) technologies and the pervasive integration of Artificial Intelligence (AI). This article has explored the multifaceted dimensions of this synergy, from the fundamental principles and enabling technologies to the key performance metrics, architectural considerations, and transformative use cases. The path ahead is one of immense opportunities, but it is also paved with significant challenges that require innovative solutions and collaborative efforts across the global telecommunications ecosystem (Khan et al., 2025; Mahesh et al., 2023).

### 10.1. Synthesizing the Role of mmWave and AI in Realizing the 6G Promise

Millimeter-wave spectrum, with its vast available bandwidth, is indispensable for achieving the terabit-per-second data rates and massive capacity envisioned for 6G. However, the inherent propagation challenges of mmWave necessitate sophisticated solutions like massive MIMO and highly adaptive beamforming. It is here that Artificial Intelligence emerges as a critical enabler, providing the intelligence to manage these complex mmWave systems effectively. AI-driven beam management, channel estimation, and interference mitigation are crucial for ensuring robust and reliable mmWave connectivity. Beyond the physical layer, AI is set to permeate every layer of the 6G architecture, from intelligent resource allocation in the RAN to predictive network slicing and automated security in the core network. This AI-native approach will transform 6G into a self-optimizing, self-healing, and self-configuring network, capable of adapting to dynamic conditions and delivering a diverse range of intelligent services with unprecedented quality of experience. The performance metrics for 6G, therefore, extend beyond traditional measures to include AI-specific indicators such as adaptability, learnability, and operational efficiency, reflecting the network’s inherent intelligence. The synergy between the raw power of mmWave and the adaptive intelligence of AI is the cornerstone upon which the ambitious vision of 6G—a vision of ubiquitous, intelligent, and immersive connectivity—will be built (Khan et al., 2025; Mahesh et al., 2023; Pennanen et al., 2024).

### 10.2. Concluding Remarks on the Transformative Potential of 6G Evolution

The evolution towards 6G, powered by mmWave and AI, holds the potential to redefine industries, enhance human capabilities, and address pressing societal challenges. From holographic communications and truly immersive XR experiences to intelligent autonomous systems and hyper-connected smart cities, the applications enabled by 6G will be transformative. However, realizing this potential requires a concerted focus on overcoming the technical hurdles related to complexity, scalability, and energy efficiency, as well as addressing the critical aspects of standardization, security, and ethical AI deployment. Future research must continue to push the boundaries of AI algorithms, mmWave hardware, and network architectures, while

fostering a global dialogue on the responsible development and governance of these powerful technologies. By charting a path that balances innovation with responsibility, the global community can harness the transformative power of AI-integrated 6G mmWave networks to create a more intelligent, connected, and sustainable future for all. The journey is complex, but the destination—a seamlessly interconnected world augmented by pervasive intelligence—is a compelling one that warrants our collective dedication and ingenuity (Singh, 2025; SNS, 2021).

## 11. Conclusion

The integration of Artificial Intelligence (AI) with 6G millimeter-wave (mmWave) technology is set to revolutionize wireless communication by enabling ultra-fast, low-latency, and highly intelligent networks. This convergence facilitates transformative applications such as immersive holographic communication, Extended Reality (XR), autonomous systems, smart city infrastructure, and advanced Industrial IoT (IIoT). AI enhances the performance, adaptability, and scalability of mmWave systems through intelligent beamforming, resource management, and real-time decision-making. Moreover, the AI-native design of 6G will allow networks to self-optimize, self-heal, and deliver personalized services with unprecedented quality of experience. While the potential is vast, realizing this vision requires addressing complex technological, operational, and ethical challenges. The convergence of millimeter-wave (mmWave) technology and artificial intelligence (AI) marks a defining milestone in the evolution toward 6G networks. This survey has illustrated how AI-driven solutions can overcome the physical and architectural challenges of mmWave systems by enabling intelligent beam management, adaptive channel estimation, and dynamic resource optimization. Together, these advancements promise to deliver the high capacity, ultra-low latency, and context-aware intelligence that characterize the envisioned 6G ecosystem. Despite its promise, this integration remains in a formative stage with notable limitations. The absence of large-scale, standardized datasets for training AI models in realistic wireless environments restricts model generalization and transferability. Additionally, computational complexity, energy consumption, and interpretability remain unresolved challenges that hinder AI deployment at the network edge and user equipment. The lack of unified frameworks for evaluating AI-centric Key Performance Indicators (KPIs)—such as adaptability and learnability—also limits objective performance benchmarking. Moreover, issues related to security, privacy, and ethical governance of data-driven wireless systems must be addressed to ensure the trustworthiness of future AI-empowered infrastructures. Future research should therefore emphasize three main directions: (1) the development of open, federated, and privacy-preserving datasets and platforms to support reproducible research; (2) the design of lightweight, explainable AI models optimized for distributed and energy-constrained network environments; and (3) the formulation of standardized methodologies for evaluating AI-native KPIs alongside conventional network metrics. Further exploration of emerging paradigms—such as semantic communications, reconfigurable intelligent surfaces, and joint sensing—communication frameworks—will also be essential in shaping a resilient and sustainable 6G landscape. In conclusion, the symbiotic evolution of mmWave communications and AI represents not just a technological transition but a paradigm shift toward networks that learn, adapt, and self-optimize. Realizing this vision will require continued interdisciplinary collaboration, rigorous experimentation, and ethical stewardship to ensure that 6G becomes a truly intelligent, inclusive, and transformative global communication fabric.

## 12. Future Work

To overcome current limitations and fully realize the vision of AI-integrated 6G mmWave networks, future research and development should focus on the following areas: **Lightweight and Energy-Efficient AI Models:** Develop new AI architectures optimized for low power consumption and real-time operation, especially at the edge and on mobile devices. **AI Standardization and Interoperability:** Collaborate globally to define common standards for AI interfaces, data formats, and performance benchmarks in 6G networks. **Secure and Privacy-Preserving AI:** Explore techniques like federated learning, differential privacy, and adversarial robustness to enhance AI security and protect user data. **Explainable and Fair AI:** Advance explainable AI (XAI) techniques to improve transparency and accountability, while ensuring fairness in resource allocation and decision-making. **AI-Driven Network Automation:** Investigate self-organizing network architectures where AI autonomously manages configuration, fault recovery, and optimization in real-time. **Cross-Layer AI Integration:** Enable seamless collaboration between AI modules across the physical, network, and application layers for end-to-end performance improvements. **Testbeds and Real-World Trials:** Establish large-scale experimental platforms to evaluate the performance, reliability, and societal impact of AI-powered 6G applications in real-world settings. Future research should focus on overcoming the above challenges through a series of targeted strategies. First, developing open, federated, and privacy-preserving datasets tailored to mmWave and hybrid 6G scenarios will enable reproducible and collaborative research while ensuring data confidentiality. Second, designing lightweight, explainable, and energy-efficient AI architectures optimized for distributed edge environments will reduce latency and improve sustainability. Third, establishing

standardized frameworks for measuring AI-native KPIs alongside traditional network metrics—such as throughput, latency, and reliability—will allow fair performance comparison and facilitate integration into emerging 6G standards. Further exploration should also extend toward novel paradigms, including reconfigurable intelligent surfaces (RIS), integrated sensing and communication (ISAC), semantic communication, and intelligent reflecting environments, where AI can dynamically coordinate resource allocation and environmental adaptation. Finally, embedding security-aware and ethically guided AI mechanisms—such as robust federated learning, adversarial defense models, and transparent decision systems—will be essential to ensure fairness, resilience, and trust in next-generation wireless networks. In summary, advancing AI–mmWave symbiosis requires not only technological innovation but also a holistic approach combining data availability, algorithmic transparency, and regulatory alignment to achieve the full vision of intelligent, self-optimizing, and human-centric 6G networks.

### 13. Limitations

Technological Complexity: Deploying and managing distributed AI across large-scale, dynamic networks is highly complex and requires robust MLOps frameworks and real-time orchestration. Scalability Issues: As device and data volumes grow, scaling AI algorithms and infrastructure efficiently remains a major challenge, particularly in edge environments. Energy Consumption: AI models, especially deep learning networks, can be computationally expensive and energy-intensive, conflicting with 6G's sustainability goals. Security Vulnerabilities: AI systems are susceptible to adversarial attacks and data manipulation, potentially undermining network performance and trust. Standardization Gaps: There is a lack of unified standards for AI integration in telecommunications, which hinders interoperability and widespread adoption. Ethical Concerns: Issues like algorithmic bias, transparency ("black box" AI), and privacy risks are critical and require proactive governance. Although the integration of millimeter-wave (mmWave) technology and artificial intelligence (AI) offers transformative potential for 6G systems, several limitations remain evident. The current research landscape lacks large-scale, realistic, and standardized datasets that capture the diverse propagation characteristics, blockage effects, and mobility patterns inherent to mmWave environments. This data scarcity limits the robustness and generalization of AI models trained under idealized or simulated conditions. Additionally, the computational complexity and energy demands of deep learning algorithms pose challenges for deployment at edge devices and user equipment, where processing and power resources are constrained. The absence of unified frameworks for assessing AI-native Key Performance Indicators (KPIs)—including adaptability, learnability, and operational efficiency—hampers consistent performance benchmarking across studies. Furthermore, data privacy, adversarial attacks, and model interpretability remain unresolved concerns that threaten both the security and trustworthiness of AI-driven mmWave systems. Ethical considerations, particularly related to autonomous decision-making and fairness in data utilization, are also insufficiently addressed. Collectively, these limitations highlight the pressing need for methodological standardization, data governance, and computational efficiency within the AI–mmWave research ecosystem.

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## A numerical scheme for singularly perturbed parabolic convection-diffusion equation using Said-Ball Polynomial

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### ABSTRACT

This study introduces a numerical approach that converges uniformly for a convection-diffusion problem with singular perturbations. The collocation approach is used, and the derivative gets interpreted in the Caputo sense. Subsequently, a numerical scheme that converges uniformly is formulated using the Said-Ball collocation technique. Then, the primary issue may be simplified to a matrix equation that relates to a set of linear algebraic equations. Following the resolution of this system, the approximation of the provided problem's unknown Said-Ball coefficients is determined. The computational result is verified to be in agreement with the theoretical expectation and to be more precise than certain established numerical methods through numerical experimentation.

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#### الخلاصة

تقوم هذه الدراسة منهجاً عددياً يقترب بانتظام لمسألة الحمل الحراري والانتشار مع الاضطرابات الشاذة. يستخدم منهج التجميع، ويفسر المشتق وفقاً لمفهوم كابوتو. بعد ذلك، تُصاغ خوارزمية عددية تقترب بانتظام ب باستخدام تقنية التجميع سعيد-بول. ثم، يمكن تبسيط المسألة الأساسية إلى معادلة مصفوفية ترتبط بمجموعة من المعادلات الجبرية الخطية. بعد حل هذا النظام، يتم تحديد تقرير معاملات سعيد-بول المجهولة لمسألة المطروحة. وقد تم التتحقق من صحة النتيجة الحاسوبية من خلال التجارب العددية، حيث تبين أنها تتوافق مع التوقعات النظرية وأنها أكثر دقة من بعض الطرق العددية المعروفة.

### 1. INTRODUCTION

The second-order one-dimensional parabolic equation, as stated in [1-4], is the primary focus of this work.

$$u_{\tau}(\zeta, \tau) - \varepsilon u_{\zeta\zeta}(\zeta, \tau) + a(\zeta)u_{\zeta}(\zeta, \tau) + b(\zeta)u(\zeta, \tau) = F(\zeta, \tau), 0 \leq \zeta \leq L, 0 \leq \tau \leq T. \quad (1)$$

where  $a(\zeta), b(\zeta)$  and  $F(\zeta, \tau)$  known real- valued functions and  $\varepsilon < 1$  is a known positive perturbation parameter that is generally taken to be close to zero. Equ. (1), known as the one- dimensional singularly perturbed convection-diffusion equation, will be considered under the initial condition (IC)

$$u(\zeta, 0) = g(\zeta), 0 \leq \zeta \leq L. \quad (2)$$

and the boundary conditions (BCs)

$$u(0, \tau) = h_0(\tau), u(L, \tau) = h_1(\tau), 0 \leq \tau \leq T, \quad (3)$$

where  $g$ ,  $h_0$  and  $h_1$ , as given by the initial and boundary conditions (2) and (3).

Consequently, various authors have developed an interest in acquiring its approximate solutions via the use of diverse numerical approaches. The convection-diffusion-reaction process consists of three distinct stages [5]. During the first stage, there is a transfer of convection and materials across different regions. In the second phase, there is a movement of diffusion and materials from an area with a high concentration to an area with a

low concentration. The last stage is a process where decay, absorption, and the interaction of substances with other components take place.

Modeling difficulties in many scientific domains, including biology, physics, and engineering, may be rather complex due to the one-dimensional parabolic convection-diffusion equation, which is a partial differential equation [6–12]. Therefore, a number of scholars have set out to find numerical solutions to these difficulties by using various numerical techniques:

A Laguerre collocation approach was suggested by Gürbüz in order to resolve the 1D parabolic convection equation in [10]. A matrix-vector equation is transformed in this technique using the provided equation and conditions. Then, by employing collocation points, the Laguerre coefficients are derived from the solution of this matrix-vector equation. Lima et al. introduced a finite difference approach in [13] for both linear and nonlinear convection-diffusion-reaction models in order to get numerical results. The authors primarily concentrate on the examination of convergence, using errors and assessing the accuracy of the procedure. The authors in [14] presented an optimum q-homotopy analysis approach for obtaining an approximate solution to the convection-diffusion problem. Additionally, the convection-diffusion-reaction has been addressed using a number of different approaches, including the following: the homotopy perturbation method [15], the finite element method [16], the Runge Kutta method [17], the Bessel collocation method [2], the weighted finite difference [18], a hybrid approximation scheme [4], and the uniform convergent numerical method [19]. The Said-Ball collocation technique is used in this investigation, where it is the first time to be used to solve singularly perturbed parabolic convection-diffusion equation.

The paper is structured as follows: The already mentioned Said-Ball polynomial is discussed in Section 2. The paper illustrates the numerical scheme in Section 3. Section 4 of the paper provides a detailed explanation of a method called residual correction, which aims to enhance an existing solution. This method can also be utilized to estimate the error of the solution. In Section 5, two numerical examples are examined to exemplify the process of residual correction and to make comparisons with other methods. Section 6 contains the final remarks regarding the paper.

## 2. Said-Ball polynomials (SBP)

In this section, we will examine how the SBP may be utilized to create the operational matrix used to solve the 2nd order one-dimensional parabolic convection-diffusion equation under consideration. SBP is one of two generalized Ball polynomials (Said-Ball and Wang-Ball) of indeterminate degree established in the '80s [20, 21], both of which have the hallmark property of strong generalization among Ball polynomials. To be more specific, the Ball polynomial was first described in [21, 22], which defines a cubic polynomial as:

$$(1-\varsigma)^2, 2\varsigma(1-\varsigma)^2, 2\varsigma^2(1-\varsigma), \varsigma^2 \quad (4)$$

according to the degree's parity, the SBP basis function of degree  $r$ , indicated by  $S_k^r(\varsigma)$ , is defined [23-27].

That is, when  $r$  is odd,  $S_k^r(\varsigma)$  is defined as

$$S_k^r(\varsigma) = \begin{cases} \binom{\frac{r-1}{2}+k}{k} \varsigma^k (1-\varsigma)^{\frac{r-1}{2}+1} & , \text{for } 0 \leq k \leq \frac{r-1}{2}, \\ \binom{\frac{r-1}{2}+r-k}{r-k} \varsigma^{\frac{r-1}{2}+1} (1-\varsigma)^{r-k} & , \text{for } \frac{r-1}{2}+1 \leq k \leq r. \end{cases}$$

when  $r$  is odd and

$$S_k^r(\varsigma) = \begin{cases} \binom{2^{-1}r+k}{k} \varsigma^k (1-\varsigma)^{2^{-1}r+1} & , \text{for } 0 \leq k \leq 2^{-1}r+1, \\ \binom{r}{2^{-1}m} \varsigma^{2^{-1}r} (1-\varsigma)^{2^{-1}r} & , \text{for } k=2^{-1}r, \\ \binom{2^{-1}r+r-k}{r-k} \varsigma^{2^{-1}r+1} (1-\varsigma)^{r-k} & , \text{for } 2^{-1}r \leq k \leq r. \end{cases}$$

when  $r$  is even.

We can write the Said-Ball curve of degree  $r$ , denoted by  $S_k^r(\varsigma)$ , with  $m+1$  control points, denoted by  $\{v_k\}_{k=0}^r$ , can be written in terms of the power basis as follows [28]

$$S(\varsigma) = \sum_{k=0}^r \sum_{l=0}^r v_k m_{k,l} \varsigma^l, 0 \leq \varsigma \leq 1 \quad (6)$$

where

$$m_{k,l} = \begin{cases} (-1)^{l-k} \binom{k + \lfloor \frac{r}{2} \rfloor}{k} \binom{\lfloor \frac{r}{2} \rfloor + 1}{l-k}, & \text{for } 0 \leq k \leq \lfloor \frac{r}{2} \rfloor, \\ (-1)^{l-k} \binom{r}{k} \binom{k}{l-k}, & \text{for } k = \frac{r}{2}, \\ (-1)^{l-\lfloor \frac{r}{2} \rfloor-k} \binom{\lfloor \frac{r}{2} \rfloor + r - k}{r - k} \binom{r - k}{l - \lfloor \frac{r}{2} \rfloor - 1}, & \text{for } \lfloor \frac{r}{2} \rfloor + 1 \leq k \leq r. \end{cases} \quad (7)$$

and  $\lfloor \zeta \rfloor$  and  $\lceil \zeta \rceil$  denote the greatest integer less than or equal to  $\zeta$  and the least integer greater than or equal to  $\zeta$  respectively

### Definition:

The Said-Ball monomial matrix is [28]

$$M = \begin{bmatrix} m_{00} & m_{01} & \cdots & \cdots & m_{0N} \\ m_{10} & m_{11} & \cdots & \cdots & m_{1N} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ m_{N0} & m_{N1} & \cdots & \cdots & m_{NN} \end{bmatrix}_{(N+1) \times (N+1)} \quad (8)$$

where  $m_{i,j}$  is given in Eq. (7)

### 3. METHOD OF SOLUTION

In this section, we will outline the procedure to be used to solve Equation (1) subject to initial and boundary conditions (2) and (3).

Firstly, we make the assumption that the solution in the truncated Said-Ball form

$$u(\zeta, \tau) \cong u_N(\zeta, \tau) = \sum_{m=0}^N \sum_{n=0}^N S_{m+1, n+1}^{m+1, n+1}(\zeta, \tau) a_{mn} \quad (9)$$

where  $S_{m+1, n+1}(\zeta, \tau) = S_{m+1}(\zeta) S_{n+1}(\tau)$  and  $u_N(\zeta, \tau)$  is the approximate solution of Eq. (1)  $a_{m,n}, m, n = 0, 1, \dots, N$ , are the unknown Said-Ball coefficients,  $N$  is chosen as any positive integer such that  $N \geq 1$ .

We can write

$$S(\tau) = X(\tau)M^T \quad (10)$$

Where  $X(\tau) = [1 \ \tau \ \tau^2 \ \dots \ \tau^N]$  and  $M$  given in Eq. (8). Then, by replacing the expression (10) into (9), we obtain the following matrix relations:

$$u_N(\zeta, \tau) = X(\zeta)M^T \bar{X}(\tau) \bar{M}^T A \quad (11)$$

where

$$\begin{aligned} \bar{X}(\tau) &= I_N \otimes X(\tau), \bar{M}^T(\tau) = I_N \otimes M^T, \\ A &= [a_{0,0} \ a_{0,1} \ \cdots \ a_{0,N} \ \cdots \ a_{N,0} \ a_{N,1} \ \cdots \ a_{N,N}]^T \end{aligned}$$

On the other hand, the relation between the matrix  $X(\tau)$  and its derivatives  $X'(\tau)$  and  $X''(\tau)$  are

$$X'(\tau) = X(\tau)\Lambda, \quad X''(\tau) = X(\tau)\Lambda^2 \quad (12)$$

where

$$\Lambda = \begin{cases} i & , j = i + 1, \\ 0 & , \text{otherwise.} \end{cases} \quad (13)$$

Next, we arrange the matrix relations of the derivatives  $u_\tau$ ,  $u_{\zeta\zeta}$  and  $u_\zeta$  by using equations (10) - (12) in the following manner.

$$\begin{aligned} u_\tau(\zeta, \tau) &= X(\zeta)M^T \bar{X}(\tau) \bar{\Lambda} \bar{M}^T A, \\ u_\zeta(\zeta, \tau) &= X(\zeta)\Lambda M^T \bar{X}(\tau) \bar{M}^T A, \\ u_{\zeta\zeta}(\zeta, \tau) &= X(\zeta)\Lambda^2 M^T \bar{X}(\tau) \bar{M}^T A, \end{aligned} \quad (14)$$

By substituting the relations (14) into Eq. (1) we have the fundamental matrix form for Eq. (1):

$$\begin{aligned} & \left\{ X(\zeta)M^T \bar{X}(\tau) \bar{\Lambda} \bar{M}^T - \varepsilon X(\zeta)\Lambda^2 M^T \bar{X}(\tau) \bar{M}^T \right. \\ & \left. + a(\zeta)X(\zeta)\Lambda M^T \bar{X}(\tau) \bar{M}^T + b(\zeta)X(\zeta)M^T \bar{X}(\tau) \bar{M}^T \right\} A = F(\zeta, \tau), 0 \leq \zeta \leq L, 0 \leq \tau \leq T. \end{aligned} \quad (15)$$

or shortly

$$WA = F \text{ or } [W; F]$$

where

$$W = X(\zeta)M^T\bar{X}(\tau)\bar{A}\bar{M}^T - \varepsilon X(\zeta)A^2M^T\bar{X}(\tau)\bar{M}^T + a(\zeta)X(\zeta)AM^T\bar{X}(\tau)\bar{M}^T + b(\zeta)X(\zeta)M^T\bar{X}(\tau)\bar{M}^T$$

By putting the collocation points, for  $\zeta \in [0, L], \tau \in [0, T]$

$$\zeta_i = \frac{1}{2} - \frac{1}{2} \cos\left(\frac{i\pi}{N+1}\right), \tau_j = \frac{1}{2} - \frac{1}{2} \cos\left(\frac{j\pi}{N+1}\right), i, j = 0, 1, \dots, N. \quad (16)$$

into Eq. (15), then we have

$$\begin{aligned} W &= [W_1 \quad W_2 \quad \dots \quad W_N]^T, \\ W_i &= [W(\zeta_i, \tau_0) \quad W(\zeta_i, \tau_1) \quad \dots \quad W(\zeta_i, \tau_N)]^T \\ G &= [G_1 \quad G_2 \quad \dots \quad G_N]^T, \\ G_i &= [G(\zeta_i, \tau_0) \quad G(\zeta_i, \tau_1) \quad \dots \quad G(\zeta_i, \tau_N)]^T, i = 0, 1, \dots, N. \end{aligned}$$

By replacing the relationship (16) in equations (2)-(3), we get the matrix representation.

$$u(\zeta, 0) = X(\zeta_i)M^T\bar{X}(0)\bar{M}^T A = g(\zeta_i)$$

for the initial condition (2) and

$$\begin{aligned} u(0, \tau) &= X(0)M^T\bar{X}(\tau_i)\bar{M}^T A = h_0(\tau_i), \\ u(L, \tau) &= X(L)M^T\bar{X}(\tau_i)\bar{M}^T A = h_1(\tau_i) \end{aligned}$$

for the boundary conditions (3), where  $i = 0, 1, \dots, N$ , or in short form

$$U_1 A = G \text{ or } [U_1; G], U_2 A = H_0 \text{ or } [U_2; H_0] \text{ and } U_3 A = H_1 \text{ or } [U_3; H_1] \quad (17)$$

In order to get the solution to equation (1) given the conditions (2)-(3), an augmented matrix was created by substituting the row matrices (15) with the  $(N+1) \times (N+1)$  rows from the matrix (17). This results in the formation of a new augmented matrix.

$$[\tilde{W}; \tilde{G}] = \begin{bmatrix} W; F \\ U_1; G \\ U_2; H_0 \\ U_3; H_1 \end{bmatrix}$$

Then we solve the system  $A = (\tilde{W})^{-1}\tilde{G}$  if  $\text{rank}(\tilde{W}) = \text{rank}(\tilde{W}; \tilde{G}) = (N+1)^2$  and  $A$  is uniquely determined. So, the coefficients of the unknown Said-Ball polynomials are determined using this method. Therefore, the solution to  $u_N(x, t)$  is approximately determined in the form of equation (9).

#### 4. ERROR ANALYSIS

The estimated error for equation (1) is provided in this section; it enhances the accuracy of the solution for the Said-Ball polynomials. The resultant equation has to be satisfied approximately, that is, for  $\zeta = \zeta_r, 0 \leq \zeta_r \leq 1$  and  $\tau = \tau_s, 0 \leq \tau_s \leq 1$ .

$$E_N(\zeta_r, \tau_s) = |u_\tau(\zeta_r, \tau_s) - \varepsilon u_{\zeta\zeta}(\zeta_r, \tau_s) + a(\zeta_r)u_\zeta(\zeta_r, \tau_s) + b(\zeta_r)u(\zeta_r, \tau_s) - F(\zeta_r, \tau_s)| \cong 0$$

Where  $E_N(\zeta_r, \tau_s) \leq 10^{-k_{rs}} = 10^{-k}$  ( $k$  is positive integer). If  $\max 10^{-k_{rs}} = 10^{-k}$  is prescribed, then the truncation limit  $N$  is increased until the difference  $E_N(\zeta_r, \tau_s)$  at each of the points becomes smaller than the prescribed  $10^{-k}$ . On the other hand, we use absolute error (AE) for measuring errors. If  $u_N(\zeta, \tau)$  is an approximation to  $u(\zeta, \tau)$  the absolute error is  $|e_N(\zeta, \tau)| = |u(\zeta, \tau) - u_N(\zeta, \tau)|$ . To facilitate the comparison of our findings with those of alternative approaches, we utilize  $L_2$  norm  $L_\infty$  and norm, which are denoted as follows:

$$\|e_N(\zeta, \tau)\|_2 = \left( \int_0^T \int_0^L (e_N(\zeta, \tau))^2 d\zeta d\tau \right)^{1/2},$$

$$\|e_N(\zeta, \tau)\|_\infty = \max_{(\zeta, \tau) \in [0, L] \times [0, T]} |e_N(\zeta, \tau)|.$$

## 5. NUMERICAL EXAMPLES

The procedure described in Section 3 is implemented on two illustrative problems in this section. Every necessary calculation has been performed using MATLAB R2021a

**Example 1.** The first example in our study is the following equation [1, 3, 4]

$$u_\tau - \varepsilon u_{\zeta\zeta} + (2\zeta + 1)u_\zeta + \zeta^2 u = \frac{e^{\zeta+\tau}}{\varepsilon}(\zeta^2 + 2\zeta + 2 - \varepsilon), \quad (18)$$

with the initial condition

$$u(\zeta, 0) = \frac{e^\zeta}{\varepsilon}, \quad 0 \leq \zeta \leq 1, \quad (19)$$

and the boundary conditions

$$u(0, \tau) = \frac{e^\tau}{\varepsilon}, \quad u(1, \tau) = \frac{e^{\tau+1}}{\varepsilon}, \quad 0 \leq \tau \leq 1. \quad (20)$$

The exact solution of the present problem is  $u(\zeta, 0) = \frac{e^{\zeta+\tau}}{\varepsilon}$ .

We have utilized the approach outlined in Section 3 to examine Example 1, considering various options for  $N$  and employing multiple values for the perturbation parameter  $\varepsilon$ . Figure 1 shows the approximate solutions  $u_6(\zeta, \tau)$  for four different  $\varepsilon$  values.

To facilitate comparison with alternative collocation methods, we have computed the  $L_2$  and  $L_\infty$  norms of the AE for  $N$  values ranging from 5 to 10. The values are presented in Table 1. While, Table 2 displays the AE for example 1, with  $N = 10$  and  $\varepsilon = 10^{-1}$ , across various values of  $\tau$ .

TABLE 1 Comparison of the  $L_\infty$  error of the AE function  $|e_N(\zeta, \tau)|$  for different values of  $N$  and in Example 1  $\varepsilon$

PM	$N = 5$	$N = 6$	$N = 7$	$N = 8$	$N = 9$	$N = 10$
$\varepsilon = 1/10$	8.4771E-04	4.4025E-06	3.0556E-07	1.3021E-08	6.2679E-10	3.3103E-11
$\varepsilon = 1/100$	8.4771E-04	5.2696E-05	1.3459E-06	3.7924E-08	1.0859E-09	3.0996E-11
$\varepsilon = 1/1000$	8.4320E-03	5.2635E-04	1.2767E-05	3.8824E-07	9.7772E-09	3.5053E-10
$\varepsilon = 1/10000$	8.4309E-02	5.2623E-03	1.2712E-04	3.8995E-06	9.7036E-08	3.4861E-09
Reff [3]	$N = 5$	$N = 6$	$N = 7$	$N = 8$	$N = 9$	$N = 10$
$\varepsilon = 1/10$	1.9640E-3	1.0855E-4	8.6060E-6	1.1654E-7	1.2083E-9	2.3913E-10
$\varepsilon = 1/100$	4.3049E-2	1.5669E-3	1.3818E-4	2.0306E-6	3.8459E-8	1.5497E-8
$\varepsilon = 1/1000$	4.7793E-1	7.1433E-2	1.1717E-2	1.9467E-4	2.2718E-6	1.2584E-7
$\varepsilon = 1/10000$	4.8544	9.8674E-1	1.6973E-1	1.1336E-2	8.2980E-5	5.1276E-6
Reff [13]	$N = 5$	$N = 6$	$N = 7$	$N = 8$	$N = 9$	
$\varepsilon = 1/10$	9.6181E-4	1.8000E-5	1.5525E-6	1.2692E-5	6.8182E-9	
$\varepsilon = 1/100$	6.0181E-3	2.2000E-4	1.1333E-5	1.1429E-7	8.5000E-8	
$\varepsilon = 1/1000$	6.3998E-2	2.1500E-3	1.1365E-4	1.3333E-6	9.2500E-7	
$\varepsilon = 1/10000$	6.5455E-1	2.1500E-2	1.1500E-3	1.3429E-5	9.0000E-6	

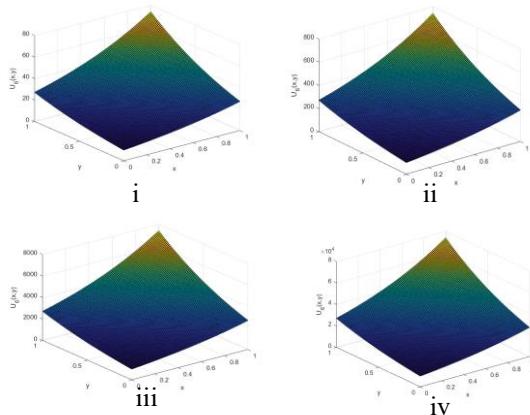


Figure 1. Approximate solutions of Example 1 obtained with  $N = 6$  corresponding to i,  $\varepsilon = 1/10$ , ii,  $\varepsilon = 1/100$ , iii,  $\varepsilon = 1/1000$  and iv,  $\varepsilon = 1/10000$ .

Table 2 Comparison the AE for example 1, with  $N = 10$  and  $\varepsilon = 10^{-1}$ , across various values of  $\tau$ .

$\varsigma_i$	$\tau = 0.1$	$\tau = 0.3$	$\tau = 0.5$	$\tau = 0.9$
0.1	9.4378E-05	1.6155E-04	1.2113E-04	3.3651E-04
0.2	1.1199E-06	1.7145E-04	3.1997E-05	2.9740E-04
0.3	3.4494E-05	1.6925E-04	6.8859E-05	2.2835E-04
0.4	3.4143E-05	1.4561E-04	1.3862E-04	1.6038E-04
0.5	2.2180E-05	9.5216E-05	1.5400E-04	8.7212E-05
0.6	7.4082E-06	3.3329E-05	1.2800E-04	1.4748E-05
0.7	1.8610E-05	7.8109E-06	1.0204E-04	2.6218E-05
0.8	6.4979E-05	4.9280E-06	1.1082E-04	7.9679E-06
0.9	1.0471E-04	5.7055E-06	1.1704E-04	2.1253E-05

**Example 2.** Next, we will address the problem that was already analyzed in references [3, 4].

$$u_{\tau} - \varepsilon u_{\varsigma\varsigma} + (2 - \varsigma^2)u_{\varsigma} + \varsigma u = 10\tau^2 e^{-\tau} \varsigma(1 - \varsigma), \varsigma, \tau \in [0,1]. \quad (21)$$

Both the initial as well as the boundary conditions could be given by:

$$\begin{aligned} u(\varsigma, 0) &= 0, \varsigma \in [0,1], \\ u(0, \tau) &= u(1, \tau) = 0, \tau \in [0,1]. \end{aligned} \quad (22)$$

Since the exact solution of this problem is not known, the residual function  $R_N(\varsigma, \tau)$  to assess the accuracy of the approximate solutions will be utilized. Example 2 is the one to which the present scheme has been applied. In Fig. 2 illustrates the residual functions of the approximate solutions obtained with different  $N$  values and for  $\varepsilon = 2^{-4}$ .

Furthermore, In figure 3, we have implemented the current technique on Example 2 using  $N = 8$  and the singular perturbation parameter values of  $\varepsilon = 2, 4, 6$ , and  $8$ . However, the data in table 3 demonstrate that the

current strategy produces outcomes that are similar to the other ways stated for this specific case. Finally, Table 4 presents the AE for example 2, considering different values of  $\tau$ ,  $N = 7$ , and  $\varepsilon = 2^{-2}$ .

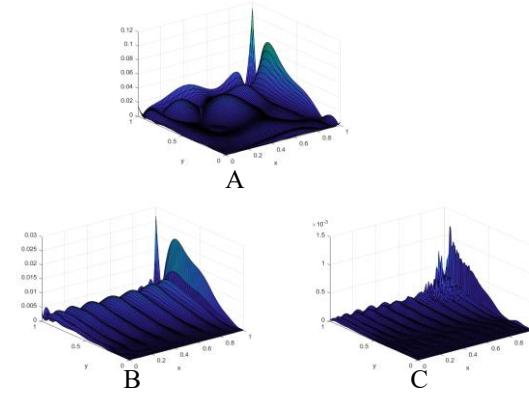


Fig 2. The residual functions of the approximate solutions for example 2, derived for A with  $N=6$ , B with  $N=10$ , and C with  $N=14$ , correspond to the selected perturbation parameter  $\varepsilon = 2^{-4}$ .

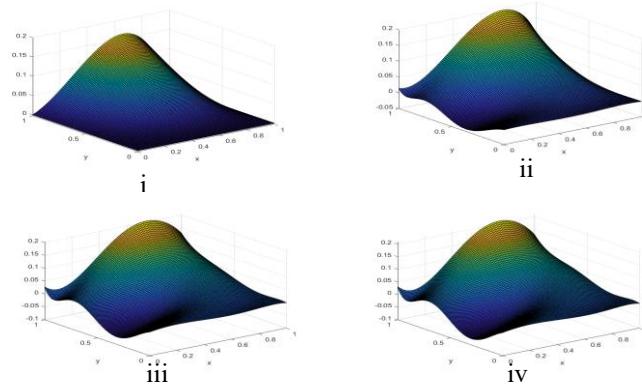


Fig. 3. Approximate solutions of Example 2 obtained with  $N=8$  corresponding to i,  $\varepsilon = 1/4$ , ii,  $\varepsilon = 1/16$ , iii,  $\varepsilon = 1/64$  and  $\varepsilon = 1/256$ .

TABLE 3. Comparison of the  $L_2$  error of the absolute error function  $|e_N(\zeta, \tau)|$  for various values of  $N$  and  $\varepsilon$  in Example 2

$\varepsilon$		$2^{-2}$	$2^{-4}$	$2^{-6}$	$2^{-8}$
PM	N=3	0. 15940E-3	0. 17052E-3	0. 17252E-3	0. 17377E-3
	N=4	0. 46406E-4	0. 82402E-4	0. 10558E-3	0. 11430E-3
Reff [3]	N=3	0. 1071E-3	0. 3357E-3	0. 8856E-3	0. 5429E-3
	N=4	0. 2723E-4	0. 2630E-3	0. 6464E-3	0. 4001E-3
Reff [2]	N=3	0. 1791E-3	0. 2454E-3	0. 4272E-3	0. 2909E-3
	N=4	0. 1090E-4	0. 1141E-3	0. 1187E-3	0. 8395E-2
Reff [29]	N=16	0. 2030E-3	0. 2810E-3	0. 3048E-2	0. 8395E-2
	N=32	0. 1113E-3	0. 1857E-3	0. 1275E-2	0. 4648E-2
Reff [30]	N=16	0. 26E-04	0. 115E-3	0. 225E-3	0. 152E-3
	N=32	0. 9921E-5	0. 51E-4	0. 167E-3	0. 144E-3
Reff [31]	N=3	0. 1124E-3	0. 1678E-3	0. 3090E-3	0. 3574E-3
	N=4	0. 6320E-4	0. 8104E-4	0. 1522E-3	0. 1934E-3

Table 4. Comparison the AE for example 2 at  $N = 7, \varepsilon = 2^{-2}$ .

$\xi_i$	$\tau = 0.1$	$\tau = 0.3$	$\tau = 0.5$	$\tau = 0.9$
0.1	1.8775E-05	2.9553E-05	2.3824E-05	2.7127E-04
0.2	5.3067E-05	6.3429E-05	2.0241E-05	6.2175E-04
0.3	1.4144E-05	2.1355E-05	3.7880E-05	1.9695E-04
0.4	6.7517E-05	5.0091E-05	2.3373E-05	5.5325E-04
0.5	6.9006E-05	3.4627E-05	7.4278E-05	4.6528E-04
0.6	2.8147E-05	4.3531E-05	6.4821E-06	3.2309E-04
0.7	1.0545E-04	6.8340E-05	1.2288E-04	6.9734E-04
0.8	2.2380E-05	1.0176E-05	6.5462E-05	2.7561E-05
0.9	1.2731E-04	5.6492E-05	2.0063E-04	6.9274E-04

## 6. Conclusions

This work presents a collocation technique that is built upon the Said-Ball approach. The method is designed to numerically solve convection-diffusion equations of parabolic type, which are often encountered in several engineering fields. The primary characteristic of the work being given is the need to solve an algebraic system of equations at each individual time step, as opposed to solving a global system produced in Said-Ball collocation techniques. The accuracy and efficiency of the suggested technique are shown by numerical tests, which are described in figures and tables. These results are compared with existing published schemes. The suggested approach can be expanded to include the fractional solutions of the singularly perturbed parabolic convection-diffusion equation.

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## Performance Evaluation of Authentication Method in Public and Private Blockchains

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### ABSTRACT

The Internet of Things (IoT) is a network of connected devices designed to perform specific tasks. Many IoT devices are lightweight, meaning they have limited storage and processing power. Because of these limitations, centralized authentication systems are often used to manage security and access control. Unfortunately, such systems suffer from limitations like single points of failure, scalability issues, cost constraints, and bottlenecks. To overcome these limitations, decentralized systems involving public and private blockchains have emerged. This research evaluates the performance of an authentication system on private (Ganache) and public (Rinkeby and Ropsten) blockchains. Ganache, is an Ethereum emulation tool that facilitates testing in private blockchains, while Rinkeby and Ropsten represent public blockchains. The evaluation metrics employed in this research are execution time, CPU usage, and memory utilization, which play a significant role in group membership association requests and data exchanges. The findings indicate that private blockchains exhibit lower time and CPU usage due to their relatively smaller number of users, whereas public blockchains demonstrate lower memory consumption in comparison.

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#### الخلاصة

إنترنت الأشياء هي مجموعة من الأجهزة المترابطة التي تهدف إلى تحقيق مهام محددة. تمتلك أجهزة إنترنت الأشياء الخفيفة قدرة تخزين ومعالجة محدودة، مما يؤدي إلى اعتماد أنظمة التوثيق المركزية. ومع ذلك، فإن هذه الأنظمة تعاني من بعض القيود مثل نقاط الفشل الواحدة، ومشاكل في التوسيع، والقيود المالية، والاختراقات. التغلب على هذه القيود، ظهرت الأنظمة اللامركزية التي تشمل البلوكشين العام والخاص. تقوم هذه الدراسة بتقييم أداء نظام التوثيق على شبكات البلوكشين الخاصة جاناش (Ganache) وال العامة رينكبي وروپستن (Rinkeby and Ropsten). جاناش هي أداة محاكاة إيثريوم (Ethereum) تسهل الاختبار في شبكات البلوكشين الخاصة، بينما تمثل رينكبي وروپستن شبكات البلوكشين العامة. تتضمن مقاييس التقييم المستخدمة في هذه الدراسة وفت التنفيذ، واستخدام وحدة المعالجة المركزية، واستهلاك الذاكرة، والتي تلعب دوراً كبيراً في طلبات ارتباط العضوية الجماعية وتبادل البيانات. تشير النتائج إلى أن شبكات البلوكشين الخاصة تبين وقئاً واستخداماً أقل لوحدة المعالجة المركزية بسبب عدد المستخدمين الأصغر نسبياً، في حين تظهر شبكات البلوكشين العامة استهلاكاً أقل للذاكرة.

## 1. INTRODUCTION

Internet of Things (IoT) is a network sensors and devices that are able to share and capture data with each other and connect together over a network [1]. One of the significant challenges preventing the widespread adoption of IoT technologies is the concerns relating to privacy and security. The evolution of IoT devices creates a new model of facilities, but at the same time it makes some security weaknesses [2]. In the time before the invention of blockchain technology, a majority of online activities were carried out through centralized servers to insure data integrity and confidentiality.

Blockchain is a decentralized database of transactions. Every user on the blockchain network maintains an authentic copy of the database. So, it is hard to add a malicious transaction because it must be

verified by all network users. A consensus mechanism ensures that all participants in a blockchain network agree on its contents. The most commonly used methods include Proof of Work (PoW), Proof of Stake (PoS), and Proof of Authority (PoA). They differ in their work style [3]. Proof of work is used by most cryptocurrency networks like Bitcoin and Litecoin. Users must prove the work to add new blocks to the blockchain. Although the mining process needs high energy consumption and processing time, proof of stake is another common one with a lower cost and lower energy consumption compared to the proof of work [4], where it depends on financial stake. Proof of work and proof of stake allow for open participation, allowing anyone to join and participate in their respective networks. However, this open participation does not exist in the proof of authority where it restricts the role of validator to trusted entities based on their trustworthiness [5].

There are three types of blockchains public, private, and federated. The public blockchains is open for all types of users to share in the network. It can be secured using crypto-economics, which is a combination of cryptographic verification and economic incentives using consensus mechanisms such as proof of work or proof of stake. Ethereum and Bitcoin, are examples of this type [6]. In private blockchains only a specific set of users has the authority to join the blockchain network. Users of this type get their permission from the organization before joining to the blockchain network. Ripple and Everledger are examples of this type [7]. The private blockchain is easier than public blockchain because the number of users is less compared to the public blockchain. Also, it offers better privacy as only users identified within the blockchain network can read the transactions [8]. The federated blockchain is a partially private blockchain. It runs under the authority of a set of organizations. So, it is a private blockchain for a specific set of organizations and it is faster and offer better scalability and privacy than a public blockchain [9].

Securing network communications is essential requirement, and one of the key measures to achieve this requirement is by properly identify devices through authentication and authorization. However, with the rapid expansion of IoT devices worldwide, traditional centralized authentication methods are becoming less effective. These methods create a single point of failure and bottlenecks, which slow down the authentication process. Studies [7, 11-13] have shown that using a single centralized server for authentication can lead to system vulnerabilities due to this single point of failure. On the other hand, there exists a decentralized authentication approach in the form of blockchain, which can be classified into two types: public blockchain and private blockchain. In public blockchain each transaction takes 14 seconds to be validated. Therefore, public blockchain is not adapted to real-time applications where the long validation time is not appropriate [14]. The private blockchain uses less power and time and is more secure than the public blockchain due to the network's authority where users being chosen [15, 16].

This research aims to evaluate the efficacy of an authentication method in public and private blockchains, specifically Rinkeby, Ropsten, and Ganache. The study investigates and compare the performance differences among these blockchains in terms of time, CPU usage, and memory consumption. This study is an extension of our previous work [24], where we primarily investigated the performance of the authentication method in public blockchains using the mentioned metrics. To the best of our knowledge, no prior studies have evaluated the performance of the public and private blockchains in context of authentication process of IoTs

## 2. RELATED WORK

Explaining research chronological, including research design, research procedure (in the form of Authentication is the process of verifying the identity of an individual by comparing his/her credentials against stored data in a database in an authentication server [17]. This process can be conducted without utilizing blockchain technology or can leverage the capabilities of a blockchain for authentication purposes. This section presents a literature review of previous studies conducted on the topic of authentication methods. The review is organized into two parts: authentication methods that do not utilize blockchain technology, and authentication methods that leverage blockchains.

### 2.1. AUTHENTICATION METHODS WITHOUT BLOCKCHAIN

Satapathy et al. [17] proposed an Internet of Things authentication method that runs on a standard Wi-Fi network and uses elliptic curve cryptography (ECC) to authenticate Internet of Things devices. The method assigns the Wi-Fi gateway to initialize system configuration and to authenticate Internet of Things devices. User's access in the method is controlled by mobile device using an Android application. However, the proposed method has the issue of using a public key, which is not effective in storage and computation for Internet of Things constrained devices. Zhang et al. [7] proposed a proximity-based authentication method between the smart phone and the Internet of Things devices. The RSS signal variation and RSS-trace are used to match the variations with the real ones. The issue with the proximity-based authentication is that the authentication data is stored on a centralized local server, resulting in a single point of failure attack. Moreover, the system requires the devices to be close enough if they want to authenticate each other.

## 2.2. AUTHENTICATION METHODS UTILIZING BLOCKCHAIN

Dorri et al. [18] proposed a lightweight, private, secure blockchain. The method uses three interrelated blockchains: private blockchain for each use case, shared private blockchain and public blockchain. It resolves the identification issue, but it has several drawbacks. Firstly, each operation produces at least eight messages, which reduces the speed of the entire system. Secondly, private blockchains are centralized, which conflicts to their principle because it limits their availability. Griggs et al. [19] proposed utilizing private blockchain to simplify secure analysis and manage a medical sensor. The system resolves many security weaknesses related to distant patient monitoring and mechanizes the transfer of announcements to all involved parties in health insurance portability and accountability. The proposed system has some drawbacks when more smart devices broadcast their transactions to several nodes waiting to confirm the next block. This is not appropriate with the healthcare system because it deals with real-time data. Fayad et al. in [20] proposed a new authentication and authorization method for IoT gateways, using both private and public blockchains. This method aims to overcome the bottleneck problem of centralized methods caused by the rapid increase in IoT devices while maintains scalable security. Private blockchain saves money over public blockchain because it does not require transaction fees. Focusing on the scalability issues in blockchain-based IoT, authors in [25] introduced a lightweight, trust-aware authentication mechanism designed to minimize storage overhead. By combining data storage optimization with homomorphic encryption for secure cloud uploading, the framework effectively balances high-performance requirements with robust security for resource-constrained devices. To eliminate the expense of digital certificates in massive IoT networks, authors in [26] introduced a blockchain-based security scheme that functions as a decentralized alternative to Certificate Authorities. This approach prioritizes confidentiality and authorization through a low-cost, methodological framework capable of managing the registration and authentication of widely distributed smart devices. Recognizing the limitations of Proof of Work in resource-constrained environments, authors in [27] proposed a lightweight blockchain system utilizing a simplified Proof of Stake (PoS) consensus and hierarchical topology. By employing efficient cryptography (ECDSA and AES-128), the framework achieved a 54% reduction in energy consumption and maintained sub-30ms latency, offering a viable alternative to traditional centralized or heavy-duty blockchain solutions. Hammi et al. [14] proposed bubbles of trust authentication method. It was executed using a public blockchain and creates secured bubbles (groups) where devices can communicate only inside each group and can't communicate outside. The method has some issues. Firstly, it is not suitable for real-time applications because it is time consuming method due to the use of public blockchain and the transaction in Ethereum is confirmed every 14 seconds (consensus needed time). Thus, transactions (messages) sent by devices will be authenticated only after this time. Secondly, there are various situations on the Internet of Things where this time is not accepted. However, the problem will be solved if a private blockchain is used.

## 3. RESEARCH METHODOLOGY

This section outlines the research methodology used in this study. The main objective is to evaluate the performance of an authentication method in secure groups within an IoT environment, where each group represents a specific application. The concept of the authentication method and secure groups is inspired by the work in [14], where an IoT group is referred to as a "bubble." In this approach, each IoT device communicates only with members of its own group and treats all other devices as potentially malicious. This ensures that the group remains secure and inaccessible to unauthorized devices.

The authentication method consists of two phases: the association phase and the data exchange phase. The association phase begins when a device attempts to join a specific group, while the data exchange phase starts when two members within the same group want to communicate. In this method, there are two types of entities: the master and the follower. The master is responsible for creating a group. When a follower wants to join, the master first verifies its credentials before granting permission. These credentials include three key values: GroupID, which identifies the group; ObjectID, which identifies the follower; and PublicAddress, which represents the follower's public address.

To join a group, the follower sends its credential values to the master using a Python socket. The master then signs the combined credential values using Node.js to generate a follower ticket on the blockchain. This ticket is verified using the Elliptic curve digital signature algorithm. If the ticket is valid, the follower becomes a member of the group. However, if the follower tries to join a group that does not exist the transaction will be canceled.

Figure 1 illustrates a dual-environment authentication framework where a Node.js backend issues signed tickets to followers for on-chain verification. The process utilizes ECDSA (ecrecover) within a blockchain environment to validate the "Association" and "Exchange" transactions against a Master public key. As shown in the figure, the architecture is implemented across both public blockchain infrastructures (using MetaMask and Rinkeby/Ropsten) and private blockchain (using Ganache and Injected Web3), with a Python Socket facilitating the communication layer between the components.

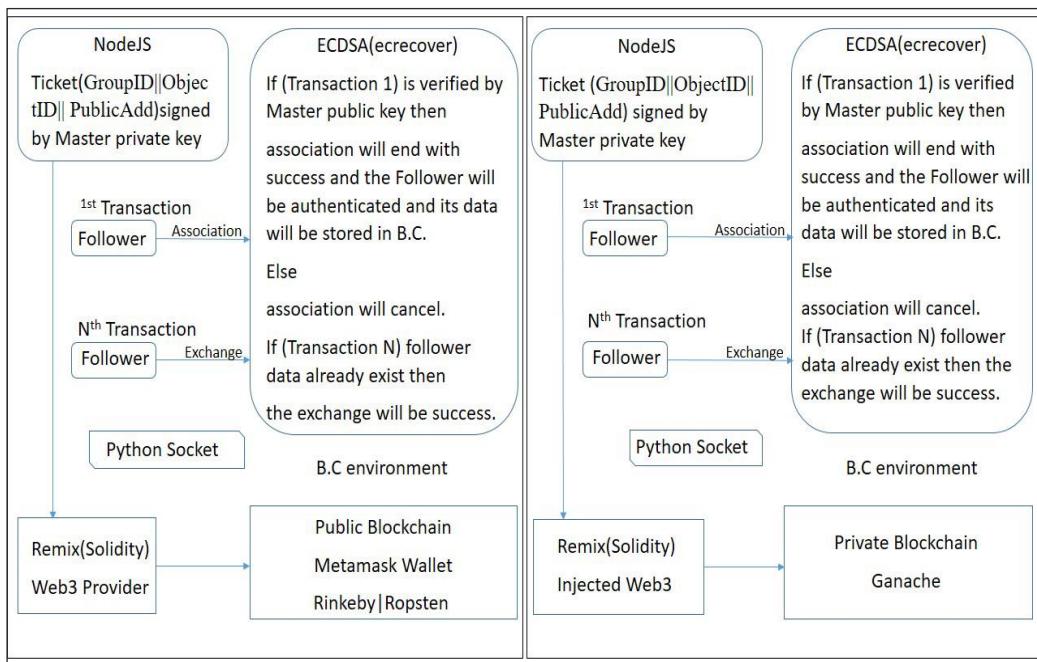


Figure 1: Authentication Method Framework

### 3.1. EVALUATION OF THE AUTHENTICATION METHOD

To evaluate the authentication method in public blockchain, two simulators were used, Rinkeby and Ropsten. The Rinkeby is a test network that uses a Proof of Authority consensus method to validate transactions. The Ropsten is a test network that uses a Proof of Work consensus method to validate transactions. The authentication method was tested using the Remix online editor with a Web3 provider environment to connect to a MetaMask wallet. The Rinkeby test network was selected, and the smart contract was deployed to it. On the Ropsten test network, the same MetaMask account was used, but test ethers were obtained by simply pressing the request button within the MetaMask account. After getting the ethers, the test network is changed to Ropsten. Finally, the same smart contract is deployed to Ropsten test network [21].

To evaluate the authentication method within private blockchain. The execution of the authentication method, along with the testing of distributed applications and smart contracts, are carried out using Ganache simulation. The authentication method is tested in Remix online editor with Injected Web3 environment to start a Ganache process. Ganache minimizes] cost associated with deploying smart contracts. When you want to deploy a smart contract on the Ethereum chain, you need to pay a gas fee for testing purposes. However, Ganache provides a solution by eliminating this cost and allowing testing smart contracts for free [22].

The construction of any group in the blockchain is made by the master of that group. The master triggers a transaction with its identifier and group identifier. The blockchain checks the uniqueness of both the group identifier and master identifier. There are two types of transactions that are performed by followers: association request transaction and data exchange transaction. In the association request transaction, if a follower wants to be a member of a specific group it sends a transaction, then the blockchain validates the uniqueness of the follower's identifier, and checks the legitimacy of the follower's ticket using the public key of the group master. If one of the conditions is not satisfied, the object cannot be a member of the group. The data exchange transaction is done by the members of any group, so a follower's ticket will not be verified because the members have already authenticated in the association request transaction.

### 3.2. EXPERIMENTAL SETUP

To evaluate the authentication method for time, CPU usage, and memory consumption, two physical devices are used. Since the authentication method has two types of entities (master and follower), the setup includes two laptops. The first laptop runs a virtual machine that acts as the master, while the second laptop has two virtual machines acting as followers. One follower runs Raspberry Pi OS (Buster version), and the other runs Ubuntu 21.04. The follower applications are developed using Python to send their credentials (GroupID, ObjectID, and PublicAddress) to the master, which then signs a ticket for authentication, Table 1 shows the specifications of the used virtual machines.

Table 1: Virtual Machine Specifications.

Virtual Machine	CPU Operation Mode	CPU Max Speed	RAM	Operating system
Master	64-bits	1.80 GHz	8.00 GB	Ubuntu 21.04
Follower 1	64-bits	1.80 GHz	4.00 GB	Ubuntu 21.04
Follower 2	32-bits	1.80 GHz	4.00 GB	Raspberry Pi OS (buster)

Rinkeby and Ropsten were used as a public blockchains and Ganache was used as a private blockchain. The smart contract that satisfies the authentication is deployed using Solidity language [23]. This study focuses on 20 investigations [24] that are conducted to evaluate the performance. The performance of the authentication method in the public and private blockchains is evaluated against the following performance metrics:

1. Time required to send an association request or data exchange and receive a response, which is a critical metric, especially for Internet of Things devices with limited storage and processing capacity. Minimizing the time consumption is crucial to optimize the performance of these devices.
2. CPU usage involved in sending an association request or data exchange and receiving a response. Minimizing CPU usage is ideal for Internet of Things devices with limited storage and processing capacity as it enhances device efficiency.
3. Memory consumption during the transmission of an association request or data exchange and receiving a response. Minimizing memory consumption is crucial for Internet of Things devices with limited storage and processing capacity, as it ensures efficient resource utilization.

#### 4. RESULTS AND DISCUSSIONS

This paper evaluates the performance of an authentication method using two public blockchains and one private blockchain. This section presents the findings from the experimental results related to time, CPU usage, and memory consumption for the both types the evaluated blockchains.

##### 4.1. TIME CONSUMPTION

Table 2, displays the average time in seconds and the corresponding standard deviation for association requests and data exchange. This metric is calculated based on 20 conducted experiments, providing a comprehensive overview of the performance metrics associated with these experiments. The analysis of Table 2 reveals that Ganache exhibits lower average time values and standard deviations compared to Rinkeby and Ropsten for both association requests and data exchange. This happens because Ganache has fewer participants in the network, resulting in faster consensus reaching. Furthermore, Ganache does not employ Proof of Work as its consensus algorithm, which eliminates the computational overhead associated with the Proof of Work method. In contrast, Rinkeby and Ropsten utilize Proof of Work, which involves extensive computation, hence leading to longer processing time. Additionally, Rinkeby and Ropsten operates as a public blockchains, accessible to a wide range of participants, which can further contribute to increased delays.

Table 2: Time Consumption.

Device Type	Association request time in seconds						Message exchange time in seconds					
	Ganache		Rinkeby		Ropsten		Ganache		Rinkeby		Ropsten	
	Avg	SD	Avg	SD	Avg	SD	Avg	SD	Avg	SD	Avg	SD
Raspberry PI	1.30	0.00	19.55	3.47	29.00	19.97	1.30	0.00	13.25	3.71	28.00	19.21
Laptop	1.30	0.00	19.07	4.06	29.00	19.97	1.30	0.00	13.55	3.71	28.00	19.21

##### 4.2. CPU USAGE

Table 3, presents the average CPU usage in seconds and the corresponding standard deviation for association requests and data exchange. This metric is calculated based on 20 conducted experiments, providing insights into the CPU usage. The results of Table 3 indicates that Ganache exhibits lower average CPU usage values and standard deviations compared to Rinkeby and Ropsten for both association requests and data exchange. This lower average is because the distinct nature of the private and public blockchains in terms of resource consumption. Rinkeby and Ropsten, being public blockchains, require substantial resources to operate and achieve network consensus. This increased resource demand contributes to higher CPU usage. Additionally, these public blockchains employ Proof of Work as their consensus algorithm, which involves solving complex mathematical puzzles. Additionally, the computational requirements of Proof of Work further contribute to the higher CPU consumption observed in Rinkeby and Ropsten. On the other hand, Ganache operates as a private blockchain limited to users within a specific organization. This user limitation base and the absence of Proof of Work as the consensus algorithm result in lower CPU usage.

Table 3: CPU Usage.

Device Type	Association request CPU usage in seconds						Message exchange CPU usage in seconds					
	Ganache		Rinkeby		Ropsten		Ganache		Rinkeby		Ropsten	
	Avg	SD	Avg	SD	Avg	SD	Avg	SD	Avg	SD	Avg	SD
Raspberry PI	8.70	2.31	9.20	4.81	15.75	10.03	7.90	2.31	8.35	4.32	11.05	5.71
Laptop	8.50	2.67	8.85	4.66	9.70	5.65	7.30	2.11	8.45	4.97	8.80	5.70

#### 4.3. MEMORY CONSUMPTION

Table 4, presents the average memory usage in kilobytes and the standard deviation for association requests and data exchange. This metric is calculated based on 20 conducted experiments. From Table 4 it is clear that Rinkeby and Ropsten has a lower memory value in average and standard deviation compared with Ganache in association requests and data exchange. This result is because Ganache is an Ethereum application, so during its running, it consumes more memory storage, but the interaction with Rinkeby and Ropsten is done using a web page that redirects to <https://etherscan.io/>.

Table 4: Memory Consumption.

Device Type	Association request memory in Kbytes						Message exchange memory in Kbytes					
	Ganache		Rinkeby		Ropsten		Ganache		Rinkeby		Ropsten	
	Avg	SD	Avg	SD	Avg	SD	Avg	SD	Avg	SD	Avg	SD
Raspberry PI	15.00	2.51	11.60	1.31	16.30	1.18	12.70	2.54	9.35	1.31	14.60	2.37
Laptop	15.50	2.87	13.15	1.18	14.05	1.15	13.50	2.80	10.90	1.29	12.05	1.15

#### 5. CONCLUSION

With the rapid spread of IoT devices and their inherent capability to communicate without human intervention, ensuring the safety and security of such communication becomes important. In this research, a performance evaluation was conducted to assess the effectiveness of an authentication method in one private and two public blockchains. The evaluation covered scenarios where IoT devices were associated with their groups and exchanged data with each other.

Based on the obtained results, it is evident that the private blockchain had lower time and CPU usage compared to the public blockchains. This was because the use of a limited number of users in the private blockchain, whereas the public blockchains are open to anyone, leading to increased number of users. However, the public blockchains demonstrated lower memory consumption compared to the private blockchain. This can be caused by the nature of public blockchains, which allow for the acceptance of a larger number of participants while efficiently managing memory resources. In the context of authentication for IoT applications, blockchain proves to be superior to centralized authentication methods by eliminating a single point of failure. However, it is important to consider the specific requirements of the IoT application. For real-time IoT applications where timing is critical, a private blockchain is recommended due to its lower time consumption. Conversely, if timing is less critical for the IoT application, a public blockchain can be chosen, as it offers the advantage of accommodating the growth number of users. The future work will involve executing a testbed to evaluate at least two IoT applications, each representing an IoT group. One of these applications focuses on real-time functionality, while the other has no strict real-time requirements. By conducting this testbed execution, we aim to evaluate the performance of the authentication method in different blockchain environments, specifically in the context of IoT applications.

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